

# MONTHLY NOTICES

OF THE

## ROYAL ASTRONOMICAL SOCIETY.

---

VOL. LXXIII.

MAY 9, 1913.

No. 7

---

Major E. H. HILLS, C.M.G., F.R.S., PRESIDENT, in the Chair.

---

Captain Francis Henry Wadsworth, R.M.S. *Caledonia*, Anchor  
Line, Glasgow,

was balloted for and duly elected a Fellow of the Society.

---

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

Nicolas Liapin, University Observatory, Odessa, Russia (proposed by F. W. Dyson).

---

Seventy-one presents were announced as having been received since the last Ordinary meeting, including, amongst others :—

Astrographic Chart ; thirty-four charts from photographs taken at Algiers, Bordeaux, and Paris, presented by the French Minister of Public Instruction.

*An Investigation on the Motion of the Stars.*

By C. V. L. Charlier, Assoc. R.A.S. (Plate 19.)

In the following is given a short summary of a research on this subject, which will be published in the next *Meddelande* from the observatory in Lund.

1. Let us consider the proper motions,  $u$  and  $v$ , in right ascension (multiplied by  $\cos \delta$ ) and in declination of the stars of a certain apparent magnitude  $m$ , occurring within a surface ( $\omega$ ), sufficiently small, at the celestial sphere. Such a surface will here be called a *square*.

Forming the correlation table of the  $u$ 's and  $v$ 's, it is easily found, even if the number of stars in the square is rather small (say 50-100), that the frequency distribution of the proper motions is far from being a normal one, the essential features of the frequency distribution being, generally, the following:—

(a) *The mean values,  $x_0$  and  $y_0$ , of  $u$  and  $v$  are different from zero.* This well-known fact is evidently due, in the main, to the motion of the Sun, relative to the stars in question.

Let O be, for every square, the origin of the co-ordinates corresponding to the values  $u=v=0$ , and let P ( $= \sqrt{x_0^2 + y_0^2}$ ) be the centre of gravity of the correlation surface.

Then OP in magnitude and direction gives the mean apparent displacement of the stars, due to the motion of the Sun.

Representing for different squares the lines OP on a sphere, it is found that these lines, prolonged, pass nearly through the same point of the sphere: *the apparent apex*.

(b) *The curves of equal frequency in the correlation table are, approximately, rather prolonged ellipses.* The centre of these ellipses coincides nearly with P. A line through P indicating the direction of the major axis of the curves of equal frequency may suitably be called the *line of symmetry* of the correlation surface.

Representing for different squares these lines of symmetry on a sphere, it is found that even these lines pass nearly through the same point of the sphere: *the apparent vertex*.

That the apparent vertex does not coincide with the apparent apex must be considered as the most important discovery made these last decenniums in that part of astronomy.

(c) *The frequency curves of the observed proper motions in right ascension as well as those in declination have a rather pronounced skew appearance.* This skew form is exhibited as well in the correlation surfaces themselves. It was this skew form, together with (b), that gave the first impulse to the *hypothesis of the two star-streams*.

(d) *These frequency curves have a rather large positive excess.* As to the meaning of this term I refer to *Meddelanden*, Ser. B, No. 4. A positive excess indicates that the stars having *small* proper motions are more numerous than should be the case if the

proper motions of the stars were distributed according to a *normal* frequency law.

2. If all the stars possessed the same luminosity, then all stars of the same *apparent* magnitude would be situated at the same distance from the Sun. In such a case the distribution of the apparent proper motions would be proportional to the distribution of the true linear velocities of the stars. The four characteristic features, (a) to (d), indicated above, were then directly applicable to the distribution of the linear velocities. Owing to the variation in the luminosities of the stars the matter stands otherwise.

The distribution of the *linear velocities* is obtained in the following way:—

I first recall the well-known fact that the characteristics of a given correlation surface, say of the variates  $x$  and  $y$ , are most easily obtained from the *mean values* of the different powers of  $x$  and  $y$ . Thus the characteristics of the observed proper motions are simple functions of the mean values— $M(u^i v^j)$ —of  $u^i v^j$  where  $i$  and  $j$  take the integer values 0, 1, 2, 3 . . . In like manner the characteristics of the linear velocities might be derived from the mean values— $M(U^i V^j)$ —of  $U^i V^j$  ( $i, j = 0, 1, 2, \dots$ ), where  $U$  and  $V$  denote the components of the linear velocities, at right angles to the line of sight, corresponding to the observed proper motions  $u$  and  $v$ . We have

$$(1) \quad U = ru, \quad V = rv,$$

if  $r$  is the distance of the star. Supposing now that  $U$  and  $V$  are *independent of  $r$* —a supposition certainly not exact, but necessary as a first approximation,—we immediately obtain

$$(2) \quad M(u^i v^j) = \theta_{i+j} M(U^i V^j),$$

where  $\theta_s$  denotes the mean value of  $1/r^s$ :

$$\theta_s = M\left(\frac{1}{r^s}\right).$$

We find from this formula that the mean values—called *moments*—of the order  $i+j$  of the linear velocities are obtained from the corresponding moments of the observed proper motions, as soon as the value of  $\theta_s$ , for  $s = i+j$ , be known. From my discussion of the number of stars of different apparent magnitudes I have derived in *Meddelanden*, Ser. B, No. 8 (here cited as 1), the formula

$$(3) \quad \theta_s = p^s q^{s^2},$$

where  $p$  and  $q$  denote certain parameters.

This formula is by no means dependent on some special assumptions made in 1. We get a similar formula from any theory hitherto discussed of the constitution of the Milky Way. Even the hypothesis of a *constant* density of the stars leads to such a value of  $\theta_s$ . The expression for  $\theta_s$  may be written:

$$\theta_s = \theta_1^s q^{s^2 - s},$$

where  $\theta_1$  denotes the mean value of  $1/r$ , a value easily determined from the now well-known amount of the velocity of the Sun.

More difficult is the determination of  $q$ . It may be obtained either from the systematic parallaxes of the stars of different magnitude or from the mean fluctuation in the absolute magnitudes. From different reasons I have found it probable that the value of  $q$  does not differ much from  $q = 1.27$ , a value adopted in the numerical calculations.

The parameter  $q$  being determined, we get, for the different squares, the moments— $M(U^i V^j)$ —of the frequency function of the linear velocities perpendicular to the line of sight from (2).

We are then able to represent the frequency distribution of the linear velocities through a series of the form

$$(4) \quad \Phi(U, V) = \phi(U, V) + \sum_{\substack{i, j=3}}^{\infty} A_{ij} \frac{\partial^{i+j} \phi}{\partial U^i \partial V^j},$$

where  $\phi(U, V)$ —the normal correlation function—has the form

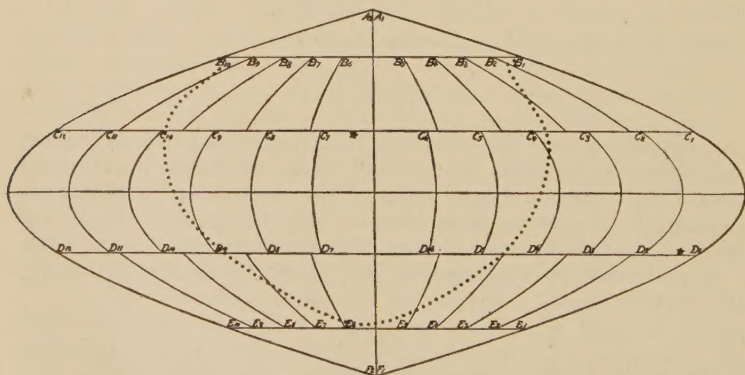
$$\phi(U, V) = \frac{K}{\sigma_1 \sigma_2 2\pi} e^{-if},$$

and

$$f = A_1(U - m_1)^2 + B_1(V - m_2)^2 + 2F_1(U - m_1)(V - m_2).$$

The values of the constants  $m_1$ ,  $m_2$ ,  $A_1$ ,  $B_1$ ,  $F_1$ , and  $A_{ij}$  are obtained directly from the moments.

The sky was divided into forty-eight "squares" of equal area in the manner described in 1. For each square the proper motions



Distribution of the forty-eight squares: projection of Flamsteed. The dots indicate the course of the Milky Way, and the two stars its poles.

from Boss's *Preliminary Catalogue* for all stars to the sixth magnitude (the magnitudes taken from Pickering's catalogues) were examined, separately for stars of the 4th and the 5th magnitude, as well as for stars brighter than  $4^m$ .

Taking together the results for the different squares, we are capable of representing the distribution of the linear velocities of the stars in space. Let the axis of  $Z$  be directed towards the true vertex (see below), and denote the velocities in this system of co-ordinates by  $U'$ ,  $V'$ ,  $W'$ , then the frequency function,  $\Phi_1$ , of the linear velocities takes the form

$$(5) \quad \Phi_1(U', V', W') = \phi_1(U', V', W') + \sum_{\substack{i+j+k=3 \\ i,j,k=0}}^{\infty} A_{ijk} \frac{\partial^{i+j+k} \phi_1}{\partial U'^i \partial V'^j \partial W'^k},$$

where

$$\phi_1 = \frac{1}{\sigma'_1 \sigma'_2 \sigma'_3 (2\pi)^{3/2}} e^{-\frac{1}{2} f'},$$

and

$$f' = \frac{(U' - m'_1)^2}{\sigma'^2_1} + \frac{(V' - m'_2)^2}{\sigma'^2_2} + \frac{(W' - m'_3)^2}{\sigma'^2_3}.$$

The first term in (5), viz.  $\phi_1$ , gives the approximate value of  $\Phi_1$ . When this term alone is taken into consideration the surfaces of equal frequency are ellipsoids. In stellar statistics the theory based on this term alone is called the *ellipsoidal hypothesis* (hypothesis of Schwarzschild). The subsequent terms give the necessary corrections to this theory.

3. The discussion of the proper motions in the forty-eight squares leads to the following conclusions arranged according to the order of the moments of the observed frequency distributions:—

(a) *Moments of the first order.*—These moments give the position of the *true apex*, and determine, moreover, an infinitesimal rotation of the system of co-ordinates, caused by the use of incorrect precessional constants or, eventually, by a *movement of the invariable plane* of the planetary system.

The *true apex* coincides with the *apparent apex* of § 1. I have determined its position separately from stars of the 4th and of the 5th magnitude, and, moreover, from all stars brighter than 6<sup>m</sup>. I give here the results obtained from the proper motions of the stars of the 5th magnitude (5<sup>m</sup>.00 to 5<sup>m</sup>.99).

$$\begin{aligned} A &= 273^\circ.17 \pm 2^\circ.06, \\ D &= +31^\circ.15 \pm 1^\circ.76. \end{aligned}$$

The determination of the apex depends on the *translatory* motion of the Sun. If the amount of this translation is supposed to be known, we are able to determine the mean parallax and the mean distance of the stars. Using the value of Campbell for the velocity of the Sun (= 20 km. per second), we get for the mean parallax of the stars of the 5th magnitude (5<sup>m</sup>.5)

$$M(\pi) = 0''.01126 \pm 0''.00072,$$

and for the mean distance

$$M(r) = 29.53 \pm 1.11 \text{ Siriometers.}^*$$

When deriving the mean value of the parallax from that of the distance (or *vice versa*), it is necessary to observe that the mean value of  $1:r$  is not the inverse of the mean value of  $r$ , but we have

$$M\left(\frac{1}{r}\right) = \frac{q^2}{M(r)} = \frac{1.6115}{M(r)}.$$

The observed mean proper motions of the stars, in different squares, indicate, in addition to the translatory movement of the system of co-ordinates, caused by the motion of the Sun, a *rotation* of the system of co-ordinates about an axis, having the spherical co-ordinates:

$$\begin{aligned} a_v &= 186.7 \\ \delta_v &= +15.1. \end{aligned}$$

This point nearly coincides with the pole of the Galaxy ( $a = 191^\circ$ ,  $\delta = +27^\circ$ ). We hence draw the conclusions: first, that the correction to the constant of precession used by Boss (that of Newcomb) is very small, if not evanescent; second, that the invariable plane moves with constant inclination to the Milky Way. The result of the numerical computation is that *the node of the invariable plane on the plane of the Milky Way has a direct motion amounting to  $0''.003528$  per year.*

( $\beta$ ) *Moments of the second order.*—These determine the position of the vertex and the mean motion of the stars in different directions.

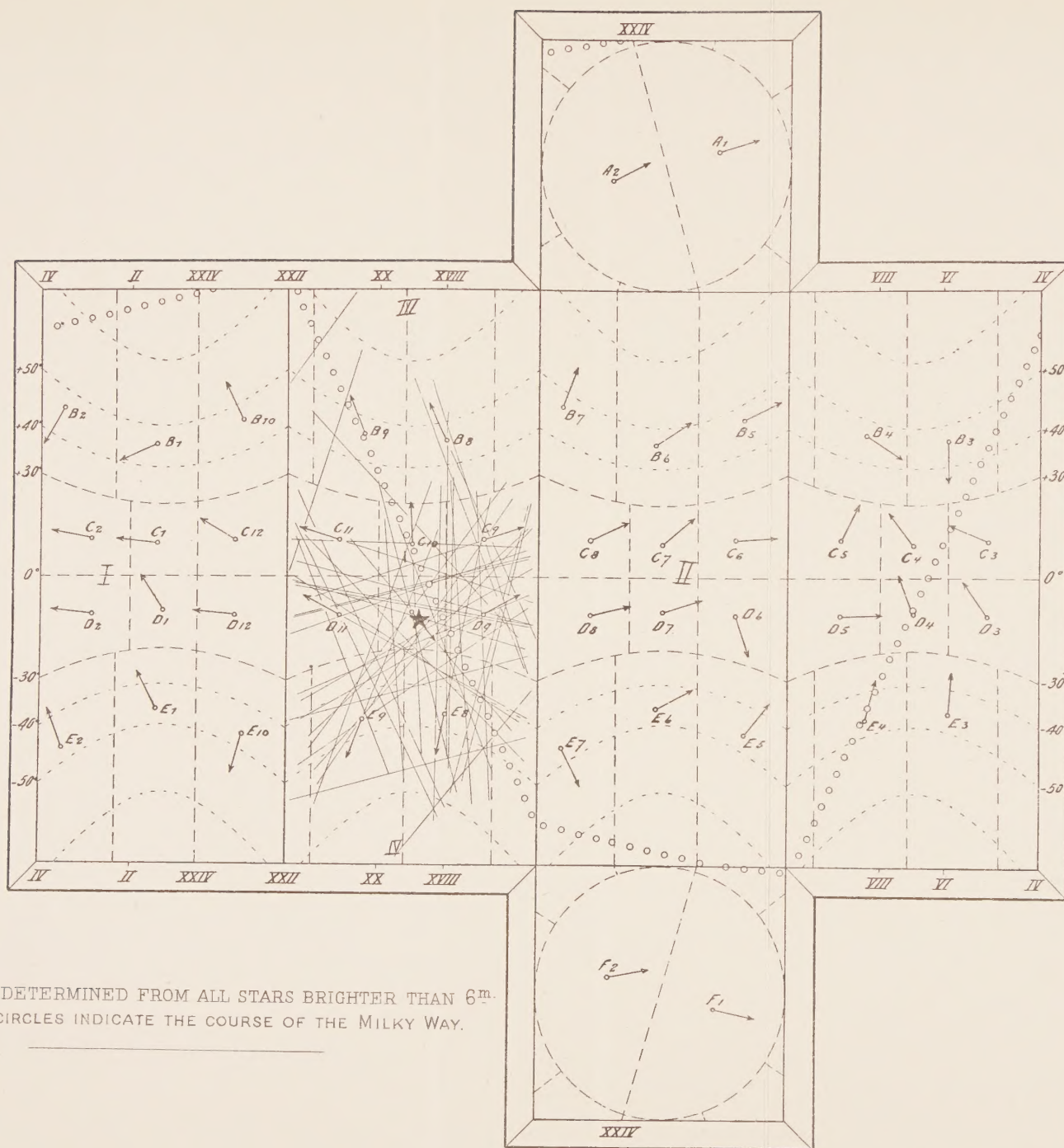
The axes of symmetry of the correlation surfaces of the linear velocities in the different squares pass, when prolonged, nearly through the same point, the *true vertex*. Its position was found to be:

$$\begin{aligned} A_v &= 18^h 51^m, \\ D_v &= -19.35. \end{aligned}$$

The true vertex does not coincide with the apparent vertex of § 1, though the difference is not large. I show in the memoir that the apex, apparent vertex, and true vertex always lie on the same great circle.

The velocity ellipsoid is found to be nearly an ellipsoid of revolution. On this supposition the *mean peculiar velocity* ( $\sigma'_1$ ) parallel to the axis of revolution of the ellipsoid and the mean

\* I have used in 1 the term "Siriometer" for denoting a distance equal to a million times the mean distance of the Sun from the Earth. I find from the *Observatory* that the Astronomer Royal has objected to this term that it "suggests a machine for measuring." I agree with him herein. But having used this term now nearly two years in my lectures, I have found that this suggestion, which I had naturally made myself, is easily obliterated.



TRUE VERTEX, DETERMINED FROM ALL STARS BRIGHTER THAN 6<sup>m</sup>.  
THE SMALL CIRCLES INDICATE THE COURSE OF THE MILKY WAY.



Digitized by the Internet Archive  
in 2026

peculiar velocity ( $\sigma'_2$ ) perpendicular to it were found to have the values:

$$\begin{aligned}\sigma'_1 &= (1.5676 \pm 0.0550)s, \\ \sigma'_2 &= (0.7921 \pm 0.0278)s,\end{aligned}$$

where  $s$  is the velocity of the Sun. Putting  $s = 20$  km./sec., we get

$$\begin{aligned}\sigma'_1 &= 31.35 \pm 1.10 \text{ km.}, \\ \sigma'_2 &= 15.84 \pm 0.56 \text{ km.}\end{aligned}$$

The eccentricity  $e$  of the velocity ellipsoid has the value  $e = 0.8626$ .

( $\gamma$ ) *Moments of the third order.*—These moments determine the *skewness* of the frequency distribution. As regarding the apparent proper motions, the skewness in the distribution of the linear velocities, for every square, is rather large and exhibits systematic variations depending on the position of a square in regard to the vertex, being generally small, and changing the sign, at vertex and antivertex. The fluctuation in the values of the coefficient of skewness is, at least in certain squares, of the same nature as a fluctuation caused by the existence of two star-streams. For testing this hypothesis I have elaborated a method for dissecting a given correlation surface into two spherical components. The problem leads to simple quadratic equations. Applying this method, it was found that the hypothesis of two star-streams does *not* account for the observed deviations from the ellipsoidal velocity distribution. For every square I obtained, indeed, *imaginary* values of the mean peculiar velocity of the stars in one of the supposed star-streams. For checking this negative result I have, for every square, introduced a system of co-ordinates with the axis of  $x$  pointing towards the vertex. If there are two star-streams, we should have a skew distribution, of the same sign, in the  $x$ -co-ordinates, whereas the  $y$ -co-ordinates should be normally distributed. The skewness was, however, found to be large in one co-ordinate as well as in the other, and, considering the  $x$ -co-ordinates alone, it was not possible to dissect their frequency curves into two normal components.

( $\delta$ ) *Moments of the fourth order.*—From these moments is obtained the *excess* of the frequency surfaces. Unlike the distribution of the apparent proper motions, which possessed a positive excess, the excess in the distribution of the linear velocities is, in almost every square, *negative*. This fact is not in accordance with the law of Maxwell (for gases), which leads, when the different mean velocities of stars with different values of the masses are taken into account, to a *positive* excess in the frequency distribution of the linear velocities. It is hence suggested that the motion of the stars in our stellar system has not yet arrived at a state of statistical equilibrium, a suggestion which is not, however, in this memoir strengthened by some mechanical considerations.

*On the Motions and Distances of the Pleiades and other Groups of Stars.* (Stellar Motions, No. 6.) By H. C. Plummer, M.A.

1. In the preceding paper of this series (p. 450) an attempt was made to exhibit the general character of the galactic motions of the stars of Type I. for which the radial velocities are known, thanks chiefly to the work of the Lick Observatory. We may now consider some of the results a little more in detail. From the Annual Report of the Mount Wilson Solar Observatory it appears that much work bearing on the present line of investigation is being carried on there with results entirely favourable to the point of view which has been maintained in these papers. Thus we find it stated (p. 174):

"(27) All local groups of stars, for which the necessary data are available, have been shown to move in space nearly parallel to the Milky Way."

And again (p. 194):—

"(4) In each of these groups the parallelism of the motion is such that parallaxes can be and were derived . . . for the great majority of the stars they must be fairly reliable. For these stars we thus know both the position in space and the amount and direction of the linear motion."

The general line of discussion initiated and followed here gains very satisfactory support from these conclusions.

*Pleiades.*

2. It is believed that no direct attempt has been made to measure the parallax of any member of the Pleiades group; nor is any indirect estimate of the distance known to me. The available data are as follows:—

TABLE I.

| P.G.C. | Star.   | Type. | <i>m</i> . | $\mu\alpha$ .        | $\mu\delta$ . | V (obs.). |
|--------|---------|-------|------------|----------------------|---------------|-----------|
| 852    | Electra | B5    | 3·8        | + '0014 <sup>s</sup> | — '050        | + 14·6    |
| 856    | Taygeta | B5    | 4·4        | + '0008              | — '048        | + 3       |
| 860    | Maia    | B5    | 4·0        | + '0021              | — '045        | + 7       |
| 865    | Merope  | B5    | 4·2        | + '0017              | — '054        | + 6       |
| 869    | Aleyone | B5    | 3·0        | + '0014              | — '048        | + 15      |
| 877    | Atlas   | B8p   | 3·8        | + '0014              | — '050        | + 13      |

We notice at once that the similarity which has long been a known property of the proper motions does not extend to the radial velocities, which fall naturally into two equal groups of three stars each. The motions of the two groups obviously cannot be reconciled by any uniform hypothesis, and the problem of determining the mean distance is therefore indeterminate. The result of applying the hypothesis of strictly galactic motion is shown in Table II.

TABLE II.

| P.G.C. | $\varpi$ . | $m_0$ . | $u$ . | $v$ . | $w$ . |
|--------|------------|---------|-------|-------|-------|
| 852    | "0444      | 7.0     | +4.8  | +10.3 | -0.9  |
| 856    | "0172      | 5.6     | -0.2  | +4.9  | -12.8 |
| 860    | "0112      | 4.0     | -4.3  | +15.1 | -16.5 |
| 865    | "0177      | 5.4     | -1.1  | +9.3  | -12.7 |
| 869    | "0429      | 6.2     | +4.8  | +10.7 | -0.7  |
| 877    | "0636      | 7.8     | +5.8  | +11.2 | +1.4  |

In no case does a negative or impossible parallax arise, but the range from ".011 to ".064 is large. The mean parallax is ".033, but the parallax corresponding to the mean distance is ".023, which still leaves a considerable difference. Table III. shows the result of assigning the two parallaxes ".02 and ".03 to the whole group.

TABLE III.

| P.G.C. | $\varpi = 0''.02$ . |       |       | $\varpi = 0''.03$ . |       |       |
|--------|---------------------|-------|-------|---------------------|-------|-------|
|        | $u$ .               | $v$ . | $w$ . | $u$ .               | $v$ . | $w$ . |
| 852    | +4.3                | +13.9 | -6.9  | +4.6                | +11.8 | -3.2  |
| 856    | -0.3                | +4.0  | -11.1 | -0.5                | +2.3  | -7.6  |
| 860    | -1.9                | +9.2  | -8.8  | -0.8                | +6.7  | -5.6  |
| 865    | -0.9                | +8.4  | -11.2 | -0.4                | +5.9  | -7.3  |
| 869    | +4.2                | +14.1 | -6.2  | +4.6                | +12.0 | -2.8  |
| 877    | +5.1                | +15.7 | -6.0  | +5.4                | +13.5 | -2.4  |
| Means  | +1.8                | +10.9 | -8.4  | +2.1                | +8.7  | -4.8  |

3. There is nothing in these figures to help us to a decision, although they do suggest that there is no very wide discrepancy in the type of motion to be inferred. Were it not for the special interest attaching to the group, we should hesitate to assign a precise distance. As it is, we fall back on our fundamental idea, supported now by the Mount Wilson results, and assume that the *mean* motion of the stars is galactic. This requires a parallax

$$\varpi = 0''.024,$$

which leads to the results shown in Table IV.

TABLE IV.

| P.G.C. | $m_0$ . | $u$ . | $v$ . | $w$ . | P.    | Q.   | N.   |
|--------|---------|-------|-------|-------|-------|------|------|
| 852    | 5.7     | +4.4  | +12.9 | -5.1  | -11.2 | +3.4 | -2.0 |
| 856    | 6.3     | -0.4  | +3.2  | -9.4  | -1.6  | +8.7 | +1.9 |
| 860    | 5.9     | -1.4  | +7.9  | -7.2  | -6.8  | +7.6 | +2.9 |
| 865    | 6.1     | -0.7  | +7.1  | -9.2  | -5.7  | +9.0 | +1.6 |
| 869    | 4.9     | +4.4  | +13.1 | -4.5  | -11.5 | +2.8 | -1.7 |
| 877    | 5.7     | +5.2  | +14.6 | -4.2  | -12.9 | +2.4 | -2.6 |
| Means  | 5.8     | +1.9  | +9.8  | -6.6  | -8.3  | +5.7 | 0.0  |

The separation into two groups persists, as it must do if we suppose the data accurate. The mean absolute magnitude ( $\overline{m} = 0''.01$ ) of the stars is about two magnitudes fainter than the general means found for their types (3.9 for B5 and 3.6 for B8). This may be due to the scattering of their light in the nebulous atmosphere which envelops them. The motions in space, corrected for the solar motion, appear to be very small, and Electra, Alcyone, and Atlas in particular have very nearly the same motion as has been found for the whole system of type A stars. Relative to the Sun the mean motion of the group is

$$12.0 \text{ km./sec. towards A} = 79^\circ, D = -33^\circ.4;$$

and though this result and the concluded parallax of the group are undoubtedly precarious, they probably represent as good approximations to the motion and distance of the Pleiades as can be drawn from the material at present available. The distance may be expressed as 130 light years, or perhaps rather greater than this.

### *Vela.*

4. The Mount Wilson report alludes to a possible local group in Vela. In all, fifteen stars belonging to this constellation have been discussed in these papers, and the material available for treatment is shown in Table V.

| P.G.C. | $\overline{m}$ . | $u$ .       | $v$ .         | $w$ .        |
|--------|------------------|-------------|---------------|--------------|
| 2226   | "0144            | -0.57+4.88x | +10.63+2.80x  | -11.09-0.44x |
| 2266   | "0087            | +8.36+6.48x | -0.25+2.30x   | +1.64-1.83x  |
| 2307   | "0114            | -3.48+5.69x | +13.88+2.38x  | -15.38-1.83x |
| 2325   | "0054            | +0.68+5.81x | +21.78+15.53x | -4.19+6.37x  |
| 2329   | "0219            | -3.78+5.20x | +14.37+3.46x  | -15.71-0.59x |
| 2341   | "0189            | -0.90+4.81x | +11.38+4.55x  | -11.40+0.30x |
| 2358   | "0164            | -5.60+5.19x | +13.48+0.92x  | -19.57-2.62x |
| 2428   | "0207            | -6.76+7.68x | +21.72+7.23x  | -18.42-0.34x |
| 2467   | "0061            | -8.41+7.37x | +22.65+7.87x  | -21.23+0.00x |
| 2526   | "0150            | -2.55+5.06x | +10.43+3.99x  | -14.94-0.92x |
| 2640   | "0230            | +7.40+5.05x | +2.92+4.47x   | +1.10-1.74x  |
| 2674   | "0131            | +0.83+5.67x | +8.83+5.84x   | -9.13-1.27x  |
| 2723   | "0343            | +0.14+7.44x | +24.34+20.56x | -4.11+3.20x  |
| 2755   | "0070            | +3.06+5.25x | +11.98+10.92x | -3.55+0.00x  |
| 2929   | (.0363)          | +6.33-0.27x | -5.03-3.12x   | +5.44-0.68x  |

This table gives the parallaxes which result from the galactic hypothesis except in the last case, where it was necessary to assign an arbitrary value. The meaning of  $x$  is given by the relation

$$x = \overline{m}/\overline{m}' - 1; \quad (x > -1);$$

where  $\varpi$  is the parallax given in the second column, and  $\varpi'$  is any other value of the parallax which it is desired to substitute for  $\varpi$ . The table, therefore, gives not only the components of velocity which result from the galactic hypothesis ( $x=0$ ), but also the readiest means of seeing the effect of assigning other distances to the stars.

5. It is sufficiently evident that all the stars cannot be reduced to one homogeneous group. At an early stage of the work the association of Nos. 2307, 2329, and 2358 was noticed, and they were assigned to the well-marked Group 3 (*L.O.B.*, 212, Table V.). To them are now added Nos. 2428 and 2467. It seems impossible to reconcile Nos. 2266, 2640, and 2929 with the rest, and these may be grouped together. There remain seven stars, among which Nos. 2226, 2341, 2526, and 2755 agree fairly well, while Nos. 2325, 2674, and 2723 can be brought into line with them by assigning to  $x$  the respective values  $-\frac{2}{3}$ ,  $+\frac{1}{2}$ , and  $-\frac{1}{2}$ . These values are purely conjectural, and any attempt at refinement would be obviously premature. The result of this adjustment and re-arrangement is shown in Table VI.

TABLE VI.

| C.          | Star.    | Type. | $m$ . | $m_0$ . | $\varpi$ . | $u$ . | $v$ . | $w$ . | P.    | Q.    | N.   |
|-------------|----------|-------|-------|---------|------------|-------|-------|-------|-------|-------|------|
| 7           | <i>e</i> | A5    | 4.1   | 4.3     | 0.114      | -3.5  | +13.9 | -15.4 | -11.6 | +16.3 |      |
| 9           | <i>u</i> | A3    | 4.8   | 6.5     | 0.219      | -3.8  | +14.4 | -15.7 | -12.2 | +16.7 |      |
| 8           | <i>a</i> | A     | 4.1   | 5.1     | 0.164      | -5.6  | +13.5 | -19.6 | -10.8 | +20.9 |      |
| 8           | ...      | B5    | 5.5   | 7.1     | 0.207      | -6.8  | +21.7 | -18.4 | -19.3 | +21.0 |      |
| 7           | ...      | B5    | 5.0   | 3.9     | 0.061      | -8.4  | +22.6 | -21.2 | -20.0 | +24.4 |      |
| Means (i)   |          |       |       |         |            | -5.6  | +17.2 | -18.1 | -14.8 | +19.9 |      |
| 6           | A        | B5    | 5.5   | 5.2     | 0.087      | +8.4  | -0.3  | +1.6  | +1.2  | -5.4  |      |
| 0           | <i>u</i> | A     | 5.3   | 7.1     | 0.230      | +7.4  | +2.9  | +1.1  | -2.1  | -4.3  |      |
| 9           | <i>i</i> | A2    | 4.6   | 7.4     | (0.363)    | +6.3  | -5.0  | +5.4  | +4.9  | -8.1  | +4.4 |
| Means (ii)  |          |       |       |         |            | +7.4  | -0.8  | +2.7  | +1.3  | -5.9  | +1.5 |
| 6           | B        | B2    | 4.9   | 5.7     | 0.144      | -0.6  | +10.6 | -11.1 | -8.6  | +10.8 |      |
| 5           | $\sigma$ | B3    | 3.7   | 4.7     | (0.162)    | -3.2  | +11.4 | -8.4  | -10.3 | +9.8  | +3.4 |
| 1           | ...      | B5    | 5.0   | 6.4     | 0.189      | -0.9  | +11.4 | -11.4 | -9.5  | +11.3 |      |
| 6           | $\kappa$ | B3    | 2.6   | 3.5     | 0.150      | -2.6  | +10.4 | -14.9 | -8.2  | +15.1 |      |
| 4           | $\phi$   | B5    | 3.7   | 3.4     | (0.087)    | +3.7  | +11.8 | -9.8  | -9.6  | +7.7  | -3.3 |
| 3           | <i>g</i> | A2    | 4.1   | 8.3     | (0.686)    | -3.6  | +14.1 | -5.7  | -13.4 | +7.8  | +4.4 |
| 5           | J        | B5p   | 4.6   | 3.8     | 0.070      | +3.1  | +12.0 | -3.6  | -10.8 | +2.6  |      |
| Means (iii) |          |       |       |         |            | -0.6  | +11.7 | -9.3  | -10.1 | +9.3  | +0.6 |

The agreement in the several divisions is not particularly good, and, on the other hand, it would be easy to find stars outside the region whose motions would accord even better. A really homogeneous group is not evident, and we must await the result

of the more special research now being carried out. The motions rather vaguely indicated above give the three convergents :—

- (i) 25.6 km./sec. towards  $A = 108^\circ$ ,  $D = -45^\circ$   
 (ii) 7.9       "       "       354       + 20  
 (iii) 15.0     "       "       93       - 38

*Perseus.*

6. In Perseus we have dealt with nine stars. Four of these, Nos. 757, 844, 894, and 986, may be omitted from the present review. For the remaining five the material for discussion may be exhibited in the same form as in Table V. above, and this is done in Table VII.

TABLE VII.

| P.G.C. | $\overline{w}$ . | $u$ .            | $v$ .            | $w$ .            |
|--------|------------------|------------------|------------------|------------------|
| 384    | 0076             | - 2'05 + 0'69x   | + 18'19 + 19'43x | - 10'81 - 7'20x  |
| 708    | 00066            | - 17'00 - 18'65x | + 53'01 + 51'33x | - 25'36 - 27'38x |
| 780    | 0022             | - 18'58 - 19'97x | + 59'86 + 58'19x | - 25'56 - 28'04x |
| 839    | 0015             | - 14'96 - 14'13x | + 32'75 + 33'90x | - 29'67 - 28'73x |
| 1003   | 0075             | - 6'28 - 5'76x   | + 21'11 + 22'16x | - 17'77 - 16'54x |

An examination of this material is specially desirable because the result of applying the galactic hypothesis ( $x=0$ ) is clearly unsatisfactory.

7. At first sight the prospect is not very promising, because all the distances must be adjusted if the stars are to be brought into line and there is no very obvious clue. However, casual inspection suggested almost immediately the successive values of  $x$ , -3, -8, -8, -6, -4. The results following from their adoption are shown in Table VIII.

TABLE VIII.

| P.G.C. | Star.   | Type. | $m$ . | $m_0$ . | $\overline{w}$ . | $u$ . | $v$ .  | $w$ .  | P.     | Q.     | N.    |
|--------|---------|-------|-------|---------|------------------|-------|--------|--------|--------|--------|-------|
| 384    | $\phi$  | Bp    | 4.2   | 4.4     | (0109)           | - 2.3 | + 12.4 | - 6.6  | - 11.5 | + 7.9  | + 3.2 |
| 708    | $\beta$ | B8    | var.  | ...     | (0033)           | - 2.1 | + 11.9 | - 3.5  | - 11.5 | + 5.0  | + 4.5 |
| 780    | ...     | B3    | 4.9   | 5.1     | (0110)           | - 2.6 | + 13.3 | - 3.2  | - 12.8 | + 5.1  | + 4.8 |
| 839    | 40      | B2    | 5.0   | 2.8     | (0037)           | - 6.5 | + 12.4 | - 12.5 | - 11.0 | + 15.1 | + 4.2 |
| 1003   | 53      | B3    | 4.9   | 5.4     | (0125)           | - 4.0 | + 12.2 | - 11.2 | - 10.8 | + 12.7 | + 2.6 |
| Means  |         |       |       |         |                  | - 3.5 | + 12.4 | - 7.4  | - 11.5 | + 9.2  | + 3.9 |

The remarkable thing is that, without premeditation, we have practically reproduced the motion of the stream Vela (iii) above, not only in the mean but also in the internal character of the movements. Relative to the Sun the resultant motion and convergent become

14.9 km./sec. towards  $A = 106^\circ$ ,  $D = -30^\circ$ .

There can be little doubt that the above parallaxes represent a material improvement. Particular interest attaches to the parallax of Algol on which the constants of the physical system

depend, and it is much to be hoped that a more special research will either establish the above value or replace it by one more certain. It may be noted that one of the four excluded stars in Perseus, 32 Persei (P.G.C. 757), may be associated with the stream Vela (ii) since we have found for this star (*L.O.B.*, 212)—

$$u = +8.2, \quad v = -4.8, \quad w = -0.5.$$

*Centaurus-Scorpio.*

8. When we come to consider the stars in Centaurus and Scorpio we naturally look for indications of the common type of motion already found to prevail largely in Vela and Perseus. Here we have a larger number of stars, twenty-five in Centaurus and fifteen in Scorpio. The search being thus restricted to a definite object, we can readily decide on twenty-four stars which suggest themselves for examination from this point of view. Table IX. gives the data in the same form as previously, so that the reader may have the readiest means of drawing an alternative conclusion if he is so minded.

TABLE IX.

| P.G.C. | $\varpi$ . | $u$ .            | $v$ .            | $w$ .            |
|--------|------------|------------------|------------------|------------------|
| 3048   | '0132      | - 2'56 + 3'90x   | + 23'81 + 22'96x | - 9'51 - 0'64x   |
| 3054   | '0248      | + 0'12 + 4'71x   | + 8'01 + 7'43x   | - 10'82 - 1'95x  |
| 3068   | '0100      | + 0'56 + 1'75x   | + 29'64 + 29'50x | - 1'27 + 0'92x   |
| 3162   | '0053      | - 4'40 + 3'11x   | + 26'73 + 26'83x | - 11'86 - 2'88x  |
| 3300   | '0176      | - 3'74 + 5'42x   | + 12'07 + 13'46x | - 16'58 - 8'96x  |
| 3452   | '0327      | - 9'01 - 8'25x   | + 49'08 + 49'34x | - 11'60 - 11'01x |
| 3521   | '0147      | + 1'73 + 2'45x   | + 11'55 + 11'86x | - 6'29 - 5'25x   |
| 3564   | '0106      | - 0'87 + 0'01x   | + 16'44 + 16'87x | - 9'29 - 8'43x   |
| 3565   | '0061      | - 4'43 + 0'85x   | + 17'26 + 19'83x | - 15'76 - 10'48x |
| 3577   | '0098      | - 7'86 - 0'77x   | + 20'68 + 24'22x | - 20'95 - 15'90x |
| 3586   | '0078      | - 1'50 + 1'01x   | + 11'39 + 12'66x | - 12'56 - 10'84x |
| 3602   | '0082      | - 0'95 - 0'55x   | + 17'22 + 17'43x | - 9'09 - 8'69x   |
| 3603   | '0160      | + 1'63 + 1'19x   | + 12'04 + 11'80x | - 6'28 - 6'77x   |
| 3615   | '0168      | 0'00 + 3'20x     | + 8'52 + 10'31x  | - 10'85 - 4'54x  |
| 3670   | '0088      | + 1'17 + 0'61x   | + 16'48 + 16'11x | - 5'34 - 6'33x   |
| 3688   | '0149      | - 1'27 + 1'62x   | + 9'88 + 11'85x  | - 12'71 - 9'87x  |
| 3724   | '0166      | + 2'27 - 0'52x   | + 13'79 + 11'66x | - 4'32 - 7'45x   |
| 3818   | '0116      | - 1'24 + 1'65x   | + 8'28 + 10'99x  | - 13'32 - 9'79x  |
| 4052   | '0099      | - 0'90 - 1'60x   | + 10'32 + 9'21x  | - 11'81 - 12'54x |
| 4119   | '0338      | + 4'39 - 0'24x   | + 9'25 + 0'87x   | - 2'05 - 3'73x   |
| 4155   | '0177      | + 1'43 - 1'37x   | + 11'18 + 5'53x  | - 7'01 - 9'81x   |
| 4218   | '0131      | - 0'05 - 1'13x   | + 9'91 + 7'30x   | - 10'35 - 11'85x |
| 4281   | '0025      | - 22'83 - 23'59x | + 44'49 + 42'24x | - 39'98 - 41'82x |
| 4439   | '0121      | + 0'03 - 0'17x   | + 9'92 + 8'58x   | - 10'20 - 11'22x |

9. In no case among the forty stars in these two constellations did the galactic hypothesis lead to a negative parallax. Nor in at least one-half of the above cases is there any need to depart from the results previously obtained. For eleven stars some alteration is necessary in order to obtain a better harmony, and the following values of  $\alpha$  have been adopted: 3048 ( $-\cdot 5$ ), 3068 ( $-\cdot 6$ ), 3162 ( $-\cdot 5$ ), 3452 ( $-\frac{3}{4}$ ), 3564 ( $-\cdot 2$ ), 3565 ( $-\cdot 3$ ), 3577 ( $-\frac{1}{3}$ ), 3602 ( $-\cdot 2$ ), 3670 ( $-\cdot 2$ ), 4119 ( $+2$ ), 4281 ( $-\frac{3}{4}$ ). With these assumptions Table X. has been constructed.

TABLE X.

| P.G.C.   | Star.      | Type. | $m$ . | $m_0$ . | $\varpi$ . | $u$ . | $v$ . | $w$ . | P.    | Q.    | N.   |
|----------|------------|-------|-------|---------|------------|-------|-------|-------|-------|-------|------|
| Centauri |            |       |       |         |            |       |       |       |       |       |      |
| 3048     | A          | B8    | 4.8   | 6.9     | (.0264)    | -4.5  | +12.3 | -9.2  | -11.3 | +11.2 | +3.9 |
| 3054     | $\lambda$  | B9    | 3.3   | 5.3     | .0248      | +0.1  | +8.0  | -10.8 | -6.1  | +10.1 |      |
| 3068     | ...        | A     | 5.3   | 7.3     | (.0250)    | -0.5  | +11.9 | -1.8  | -11.5 | +2.7  | +3.9 |
| 3162     | ...        | B5    | 4.8   | 4.9     | (.0106)    | -6.0  | +13.3 | -10.4 | -12.2 | +13.0 | +4.5 |
| 3300     | $l$        | B8    | 4.8   | 6.0     | .0176      | -3.7  | +12.1 | -16.6 | -9.8  | +17.4 |      |
| 3452     | $i$        | A2    | 2.9   | 8.5     | (.1308)    | -2.8  | +12.2 | -3.3  | -11.9 | +5.2  | +4.1 |
| 3521     | $\epsilon$ | B1    | 2.6   | 3.4     | .0147      | +1.7  | +11.6 | -6.3  | -10.1 | +5.6  |      |
| 3564     | $\nu$      | B2    | 3.5   | 4.1     | (.0133)    | -0.9  | +13.1 | -7.6  | -11.7 | +8.1  | +1.4 |
| 3565     | $\mu$      | B2p   | 3.3   | 3.0     | (.0087)    | -4.7  | +11.3 | -12.6 | -9.7  | +14.2 | +2.8 |
| 3577     | $k$        | B5    | 4.7   | 5.5     | (.0147)    | -7.6  | +12.6 | -15.6 | -10.9 | +18.3 | +3.7 |
| 3586     | $h$        | B5    | 4.8   | 4.3     | .0078      | -1.5  | +11.4 | -12.6 | -9.4  | +12.7 |      |
| 3602     | $\phi$     | B3    | 4.0   | 4.1     | (.0103)    | -0.8  | +13.7 | -7.4  | -12.3 | +8.0  | +1.3 |
| 3603     | $v^1$      | B3    | 4.2   | 5.2     | .0160      | +1.6  | +12.0 | -6.3  | -10.6 | +5.6  |      |
| 3615     | $\beta$    | B1    | 0.9   | 2.0     | .0168      | 0.0   | +8.5  | -10.8 | -6.6  | +10.2 |      |
| 3670     | $v$        | B5    | 4.4   | 4.6     | (.0110)    | +1.0  | +13.3 | -4.1  | -12.3 | +4.2  | +1.2 |
| 3688     | $\alpha$   | B5    | 4.6   | 5.5     | .0149      | -1.3  | +9.9  | -12.7 | -7.9  | +12.5 |      |
| 3724     | $\eta$     | B3p   | 2.6   | 3.7     | .0166      | +2.3  | +13.8 | -4.3  | -12.5 | +3.8  |      |
| 3818     | $\kappa$   | B3    | 3.4   | 3.7     | .0116      | -1.2  | +8.3  | -13.3 | -6.2  | +13.0 |      |
| Scorpii  |            |       |       |         |            |       |       |       |       |       |      |
| 4052     | $\rho$     | B3    | 4.0   | 4.0     | .0099      | -0.9  | +10.3 | -11.8 | -8.4  | +11.5 |      |
| 4119     | $\psi$     | A2    | 4.9   | 5.2     | (.0113)    | +3.9  | +11.0 | -9.5  | -8.8  | +7.4  | -3.2 |
| 4155     | 19         | A3    | 4.8   | 6.0     | .0177      | +1.4  | +11.2 | -7.0  | -9.7  | +6.3  |      |
| 4218     | $\tau$     | B     | 2.9   | 3.5     | .0131      | -0.1  | +9.9  | -10.3 | -8.1  | +9.9  |      |
| 4281     | $\mu^2$    | B2    | 3.6   | 3.6     | (.0100)    | -5.1  | +12.8 | -8.6  | -11.8 | +10.9 | +4.0 |
| 4439     | $\lambda$  | B2    | 1.7   | 2.1     | .0121      | 0.0   | +9.9  | -10.2 | -8.1  | +9.7  |      |
| Means    |            |       |       |         |            | -1.2  | +11.4 | -9.3  | -9.9  | +9.6  | +1.2 |

Thus the means for twenty-four stars in Centaurus and Scorpio give for the convergent:

14.8 km./sec. towards A =  $96^\circ$ , D =  $-39^\circ$ ,  
as compared with the result from seven stars in Vela,

15.0 km./sec. towards A =  $93^\circ$ , D =  $-38^\circ$ ,  
and the result from five stars in Perseus,

14.9 km./sec. towards A =  $106^\circ$ , D =  $-30^\circ$ .

This agreement lends a certain confirmation to the motion assigned to the latter group. It may again be noted that here also two of the excluded stars seem to share the peculiar motion of the small group Vela (ii), for we have the results

|             |                   |            |            |            |
|-------------|-------------------|------------|------------|------------|
| P.G.C. 3352 | <i>n</i> Centauri | $u = +8.0$ | $v = -8.7$ | $w = -2.5$ |
| 4334        | <i>k</i> Scorpii  | $+8.2$     | $-6.4$     | $-1.1$     |

### *Conclusion.*

10. This examination of the three regions mentioned in the Mount Wilson report, Vela, Perseus, and Centaurus-Scorpio, has had the effect of bringing the results into better general harmony on the one hand, but of leaving an undeniable measure of uncertainty on the other. The group in Vela has been subdivided into three parts more or less distinct. Some order has been evolved out of the evident irregularity in Perseus. In Centaurus-Scorpio the number of stars apparently possessing a certain type of motion has been notably increased. But in spite of these apparent improvements and a consistency which at first sight is as satisfactory as can be expected, there remains an element of doubt when we try to advance from a general qualitative truth to results of a higher order of precision. It may be that the Mount Wilson discussion applies to an entirely different set of stars, and it would be idle to forecast its outcome before publication, which will be awaited with interest. The material in the present paper relates mainly to stars of very early type. Here the difficulty of isolating local groups with certainty is very great. The result of the present paper is, indeed, in precisely the opposite direction. So far from separating distinct types of motion, it has had the effect of emphasising a substantial unity. It is at least doubtful if local groups exist at all, if thereby is intended a type of motion which exists locally and not elsewhere. On the other hand, a few predominant types of motion of practically universal extent do exist, and the difficulty is to decide whether any particular star is to be assigned to one type or another. Thus, for example, the star 53 Persei might equally well be assigned to the group Vela (i) or Vela (iii), and the stars *l* and *k* Centauri ought probably to be removed from Table X. and transferred to the former of these groups. No large component normal to the plane of the Milky Way has shown itself, and the approximate truth of the galactic hypothesis for a considerable fraction of the early type stars in this sense can hardly be questioned. But it seems fairly clear that any material advance towards more precise and certain results will require more data than are at present available and the most critical examination.

*Dunsink Observatory :*  
1913 May 3.

*Observations of the Variable Star 97·1910 Cygni,*1900·0  $\alpha$  19<sup>h</sup> 49<sup>m</sup> 55<sup>s</sup>·01  $\delta$  +36° 46' 57"·4.

By E. E. Barnard.

The discovery of this star was announced in *A.N.* 4448, pp. 126-128, by Mr. A. R. Hinks, who found it on three plates taken at Cambridge (England) on 1909 August 7, 10, and 12. It was not visible later on plates taken in 1910.

At first it was thought the star might be a nova, and on this



Chart of the region of the Variable Star 97·1910 Cygni.  
The Variable Star is shown at its brightest.

assumption I began a series of measures of it with respect to small stars near it—a custom which I have followed for some years with regard to the novæ. On examining the object on March 19, after it had passed to the morning sky, I found it to be as bright as the 11th or 12th magnitude, which showed that it was probably simply a variable star of long period. It was thought best, however, to

continue the measures, as they will be valuable in the future. The region is rich in small stars, and as the task would be an unnecessary one, I have not measured all the stars that could be seen in the field.

The observations seem to show that the period is about nineteen or twenty months, with a range of some six magnitudes in brightness. The normal condition of the star is faint. At its minimum it seems to pass beyond reach, visually, of the 40-inch telescope. The light of the small stars near makes the variable specially difficult when faint. The observations of its colour at maximum are somewhat discordant, but it is perhaps slightly yellowish.

I have collected here the micrometer measures of position and the estimation of its brightness. These observations will doubtless give some idea of its period and light curve, especially when the magnitudes of the comparison stars have been accurately determined. The magnitudes assigned these stars are on the same scale as that used by me in my measures of the faint stars in the various clusters, but I do not feel sure that they are sufficiently accurate to determine the light curve.

In the early observations the stars 20 and 21 were seen as one star and assigned a number. In this paper I have indicated this condition by the combined number (20, 21). The measures of these two stars are uncertain.

When I first sought the star on the first of November 1910 it was very faint, and there was some uncertainty of its identity because of the presence of several similar stars near it. It was finally located with certainty by means of the micrometer. The observations seem to show that it must then have been slowly recovering from minimum, which it had passed some time earlier in the year.

At first there was some uncertainty as to the exact position of the variable. Taking the place of star 30 as a good starting-point, I determined the position of that star with the aid of Lund A.G.C. 8805 (which is a double star) on 1910 November 6.

$$\begin{array}{l} \text{Star 30-Lund 8805.} \\ \Delta\alpha + 0^m 41^s.58 \text{ (10 tr.)} \quad \Delta\delta - 1' 8''.4 \text{ (3).} \end{array}$$

The comparisons were made with the north component of 8805. This gives for the place of star 30,

$$1910.0 \quad \alpha 19^h 50^m 23^s.39 \quad \delta + 36^\circ 48' 54''.7.$$

As there were several faint stars near the supposed place of the variable, one of these (22) was connected with star 30.

$$\begin{array}{l} \text{Star 22-Star 30.} \\ \Delta\alpha \cos \delta - 65''.8 \text{ (5)} = \Delta\alpha - 5^s.48 \quad \Delta\delta - 0' 22''.8 \text{ (4).} \end{array}$$

These gave for the position of star 22

$$1910.0 \quad \alpha 19^h 50^m 17^s.91 \quad \delta + 36^\circ 48' 31''.9.$$

The final measures of the relative positions of the variable and star 30 give for the variable the position

$$1910.0 \quad \alpha 19^h 50^m 16^s.77 \quad \delta + 36^\circ 48' 28''.9.$$

This differs from Mr. Hinks's place by  $0^{\text{s}}.10$  in right ascension and by  $1''.1$  in declination. This difference is doubtless due to my comparison star being double.

On 1910 November 13 and 24 the following rough measures, of one setting each, were made for the positions of some of the stars in the field:—

| Stars.      | $\Delta\alpha \cos \delta.$ | $\Delta\delta.$ |
|-------------|-----------------------------|-----------------|
| 2-30        | -236".7                     | "               |
| 3-30        | -214".6                     |                 |
| 6-30        | -157".8                     |                 |
| 11-30       | -120".0                     | +11'.9          |
| 14-30       | -108".4                     |                 |
| 33-30       | +73'.5                      | -85'.2          |
| 34-30       | +111'.2                     | -25'.0          |
| 19-30       | -79'.9                      | -69'.2          |
| 16-30       | -88'.5                      | -48'.9          |
| 23-30       | -60'.4                      | -6'.7           |
| (20, 21)-30 | -72'.1                      | -16'.4          |
| 12-30       | ...                         | -6'.2           |

The following measures of the Lund star (8805), which is Burnham G.C. 9714, were made:—

|          |         |           |       | Mags |      |
|----------|---------|-----------|-------|------|------|
| 1910'931 | Dec. 6  | 178°.83   | 2".08 | 8    | 9    |
| '944     | 11      | [182°.36] | 2'.12 |      |      |
| 1911'213 | Mar. 19 | 178°.46   | 2'.13 |      |      |
| '372     | May 16  | 179°.01   | 2'.05 | 9'.0 | 9'.7 |
| '462     | June 18 | 179°.28   | 2'.04 |      |      |
| 1911'184 |         | 178°.89   | 2'.08 | 8'.5 | 9'.3 |

The position angle of December 11 is rejected. A note says that when the wires were set  $90^\circ$  different, for the distances, it was noticed that they were not perpendicular to the line between the stars. They were then set perpendicular by estimation, and the distance measures made. The resulting position angle would have been  $178^\circ.4$ .

The following are the measures of the variable and some of the stars near it:—

|          |         |         |         | 6 and 1. | Mags. |                                     |
|----------|---------|---------|---------|----------|-------|-------------------------------------|
| 1911'640 | Aug. 22 | 208°.52 | 172".82 | ...      | 12'.7 |                                     |
| '659     | 29      | 208°.50 | 173'.28 | ...      | 12.5  | { Very faint. Lost in clouds.       |
| '667     | Sept. 1 | 208°.39 | 173'.26 | ...      | ...   | { Seeing very poor.                 |
| '673     | 3       | 208°.50 | 172'.89 | ...      | 11'.7 | { Very high wind shaking telescope. |
| 1911'660 |         | 208°.48 | 173'.06 | ...      | 12'.3 |                                     |

6 and 2.

|          |       |        |       | Mags. |      |
|----------|-------|--------|-------|-------|------|
| 1911'333 | May 2 | 302°71 | 94"14 | ...   | 12·8 |
| '391     | 23    | 302°40 | 93'73 |       |      |
| '404     | 28    | 302°73 | 93'79 |       |      |
| <hr/>    |       | <hr/>  |       | <hr/> |      |
| 1911'376 |       | 302°61 | 93°89 | ...   | 12·8 |

6 and 4.

|          |         |        |        | Mags. |      |                                           |
|----------|---------|--------|--------|-------|------|-------------------------------------------|
| 1911'640 | Aug. 22 | 193°15 | 191"14 | ...   | 11   |                                           |
| '659     | 29      | 193°15 | 191°68 | ..    | 11   |                                           |
| '667     | Sept. 1 | 193°22 | 191°47 | ...   | ...  |                                           |
| '673     | 3       | 193°27 | 191°54 | ...   | 11'3 |                                           |
| <hr/>    |         | <hr/>  |        | <hr/> |      | Very high wind<br>shaking tele-<br>scope. |
| 1911'660 |         | 193°20 | 191°46 | ...   | 11'1 |                                           |

6 and 5.

|          |        |        |        | Mags. |      |                   |
|----------|--------|--------|--------|-------|------|-------------------|
| 1911'270 | Apr. 9 | ...    | 106°86 |       |      |                   |
| '314     | 25     | 195°28 | 106°44 | ...   | 13°6 |                   |
| '333     | May 2  | 195°44 | 107°07 | ...   | 13°5 |                   |
| '372     | 16     | 194°98 | 107°04 | ...   | ...  |                   |
| <hr/>    |        | <hr/>  |        | <hr/> |      | V. faint in haze. |
| 1911'322 |        | 195°23 | 106°85 | ...   | 13°6 |                   |

Var. and 2.

|          |         |        |        | Mags. |     |                |
|----------|---------|--------|--------|-------|-----|----------------|
| 1911'372 | May 16  | 281°75 | 160"57 | ...   | ... |                |
| '468     | June 20 | 281°77 | 159°75 |       |     |                |
| 1912'686 | Sept. 7 | 281°94 | 160°55 |       |     |                |
| '694     | 10      | 281°64 | 160°34 |       |     |                |
| <hr/>    |         | <hr/>  |        | <hr/> |     | Faint in haze. |
| 1912'055 |         | 281°78 | 160°30 |       |     |                |

Var. and 6.

|          |         |        | Var. and C. | Mags.                                                  |                                                               |
|----------|---------|--------|-------------|--------------------------------------------------------|---------------------------------------------------------------|
| 1911'213 | Mar. 19 | 257°30 | 80"19       | <div> <div></div> <div>...</div> <div>...</div> </div> | <div> <div>Faint in thick,</div> <div>misty sky.</div> </div> |
| '232     | 26      | 257°40 | 80°16       |                                                        |                                                               |
| '270     | Apr. 9  | 257°60 | 79°85       |                                                        |                                                               |
| '290     | 16      | 257°37 | 80°02       |                                                        |                                                               |
| '309     | 23      | 257°19 | 79°73       |                                                        |                                                               |
| <hr/>    |         | <hr/>  | <hr/>       |                                                        |                                                               |
| 1911'263 |         | 257°37 | 79°99       |                                                        |                                                               |

Var. and 11.

|          |         |        |       | Mags. |      |             |
|----------|---------|--------|-------|-------|------|-------------|
| 1911'213 | Mar. 19 | 313°11 | 55"48 | ...   | 13°8 |             |
| '232     | 26      | 312°85 | 55°77 | ...   | ...  |             |
| '270     | Apr. 9  | 312°38 | 55°76 |       |      |             |
| '290     | 16      | 312°68 | 55°72 |       |      |             |
| '309     | 23      | 312°75 | 55°34 |       |      | Very faint. |
| '462     | June 18 | 312°88 | 55°54 |       |      |             |
| <hr/>    |         | <hr/>  |       | <hr/> |      |             |
| 1911'296 |         | 312°78 | 55°60 | ...   | 13°8 |             |

## Var. and 12.

|          |         |        |       | Mags. |    |
|----------|---------|--------|-------|-------|----|
| 1911'314 | Apr. 25 | 299°29 | 42"13 | ...   | 14 |
| 1912'667 | Aug. 31 | 298°76 | 42"14 |       |    |
| '669     | Sept. 1 | 298°81 | 42'07 |       |    |
| <hr/>    |         | <hr/>  |       | <hr/> |    |
| 1912'217 |         | 298°95 | 42'11 | ...   | 14 |

## Var. and 14.

|          |         |        |       | Mags. |      |
|----------|---------|--------|-------|-------|------|
| 1911'232 | Mar. 26 | 198°51 | 94'84 | ...   | 13'5 |
| '270     | Apr. 9  | 198°56 | ...   |       |      |
| '290     | 16      | 198°28 | 95'04 |       |      |
| '309     | 23      | 198°40 | 94'98 |       |      |
| <hr/>    |         | <hr/>  |       | <hr/> |      |
| 1911'275 |         | 198°44 | 94'95 | ...   | 13'5 |

## 14 and 13.

|          |        |        |       | Mags. |      |
|----------|--------|--------|-------|-------|------|
| 1911'424 | June 4 | 196°68 | 17"69 | ...   | 14   |
| 1912'426 | June 4 | 195°92 | 17'72 | ...   | 13'5 |
| '434     | 7      | 197°79 | 17'77 | ...   | ...  |
| '475     | 22     | 196°37 | 17°59 |       |      |
| <hr/>    |        | <hr/>  |       | <hr/> |      |
| 1912'190 |        | 196°69 | 17°69 | ...   | 13'8 |

{ Faint in clouds.  
A fainter star  
near 13 and  
south

## Var. and 15.

|          |         |        |       | Mags. |      |
|----------|---------|--------|-------|-------|------|
| 1911'314 | Apr. 25 | 221°57 | 22'16 | ...   | 14'5 |
| '333     | May 2   | 223°57 | 22°29 | ...   | 14'7 |
| '391     | 23      | 222°20 | 22'49 | ...   | 13'5 |
| <hr/>    |         | <hr/>  |       | <hr/> |      |
| 1911'346 |         | 222°45 | 22'31 | ...   | 14'2 |

## Var. and 16.

|          |         |        |       | Mags. |      |
|----------|---------|--------|-------|-------|------|
| 1911'314 | Apr. 25 | 203°87 | 23"87 | ...   | 14   |
| '333     | May 2   | 203°18 | 24'34 | ...   | 14'5 |
| '391     | 23      | 202°00 | 24'21 | ...   | 13'3 |
| '404     | 28      | 200°74 | 24'40 | ...   | ...  |
| <hr/>    |         | <hr/>  |       | <hr/> |      |
| 1911'360 |         | 202°45 | 24'20 | ...   | 13'9 |

V. faint in clouds.

## Var. and 19.

|          |         |        |       | Mags. |      |
|----------|---------|--------|-------|-------|------|
| 1911'314 | Apr. 25 | 180°63 | 43"03 | ...   | 13'8 |
| '333     | May 2   | 181°64 | 43°29 | ...   | 13'6 |
| '391     | 23      | 179°93 | 43'30 | ...   | 13'3 |
| 1912'667 | Aug. 31 | 180°66 | 43'43 |       |      |
| '669     | Sept. 1 | 180°87 | 43°59 |       |      |
| <hr/>    |         | <hr/>  |       | <hr/> |      |
| 1911'875 |         | 180°75 | 43'33 | ...   | 13'6 |

## Var. and (20, 21).

|          |        |       |        | Mags. |    | } | Single distances.<br>Possibly two<br>faint stars at<br>this point. |
|----------|--------|-------|--------|-------|----|---|--------------------------------------------------------------------|
|          |        |       |        |       |    |   |                                                                    |
| 1911.424 | June 4 | 38°69 | 14''06 | ...   | 16 |   |                                                                    |

## 22 and 20.

|          |         |        |       | Mags. |    |
|----------|---------|--------|-------|-------|----|
| 1912.612 | Aug. 11 | 317°48 | 9''92 | ...   | 16 |

## 22 and 21.

|          |         |        |       | Mags. |    |
|----------|---------|--------|-------|-------|----|
| 1912.612 | Aug. 11 | 344°48 | 9''22 | ...   | 16 |

## Var. and 22.

|          |         |       |        | Mags. |      |                                      |  |
|----------|---------|-------|--------|-------|------|--------------------------------------|--|
|          |         |       |        |       |      |                                      |  |
| 1911.314 | Apr. 25 | 73°77 | 14''53 | ...   | 15   |                                      |  |
| 391      | May 23  | 75°02 | 14'32  | ...   | 15   |                                      |  |
| 462      | June 18 | 75°22 | 14'37  | ...   | 14   |                                      |  |
| 821      | Oct. 27 | 76°67 | 14'27  |       |      |                                      |  |
| 1912.426 | June 4  | 72°57 | 15°05  | ...   | ...  | Single distances.<br>1 setting each. |  |
| [ 532    | July 13 | 78°10 | 13'30] | ...   | ...  |                                      |  |
| 667      | Aug. 31 | 76°66 | 14'13  |       |      |                                      |  |
| 1911.847 |         | 74°98 | 14'44  | ...   | 14'7 |                                      |  |

## Var. and 23.

|           |        |       |         | Mags. |     |                                                 |  |
|-----------|--------|-------|---------|-------|-----|-------------------------------------------------|--|
|           |        |       |         |       |     |                                                 |  |
| [1911.424 | June 4 | 41°59 | 26''87] | ...   | 15  | Single distances.<br>{ Faint and<br>blurred.    |  |
| 1912.686  | Sept 7 | 43°14 | 26°97   | ...   | ... |                                                 |  |
| 689       | 8      | 43°80 | 26°62   |       |     |                                                 |  |
| 694       | 10     | 43°55 | 26°90   | ..    | ... | { Very faint in<br>thick sky.<br>23 very faint. |  |
| 713       | 17     | 43°86 | 26°83   | ...   | ... |                                                 |  |
| 1912.696  |        | 43°59 | 26°83   | ..    | 15  |                                                 |  |

## Var. and 26.

|          |         |       |        |
|----------|---------|-------|--------|
| 1912.667 | Aug. 31 | 76°50 | 59''41 |
| 669      | Sept. 1 | 76°36 | 59°44  |
| 686      | 7       | 76°20 | 59°52  |
| 1912.674 |         | 76°35 | 59°46  |

## 30 and Var.

|          |         |        |        | Mags. |    |
|----------|---------|--------|--------|-------|----|
|          |         |        |        |       |    |
| 1911.213 | Mar. 19 | 252°18 | 83''66 | 11'5  | 11 |
| 232      | 26      | 251°95 | 83°48  |       |    |
| 270      | Apr. 9  | 252°07 | 83°60  |       |    |
| 290      | 16      | 251°86 | 83°83  |       |    |
| 309      | 23      | 252°01 | 83°57  |       |    |
| 1911.263 |         | 252°01 | 83°63  | 11'5  | 11 |

These give  $\Delta\alpha - 0^m 6^s.62$   $\Delta\delta - 0' 25''.8$ .

## 30 and 2.

|           |         |        |         | Mags. |      |                                |
|-----------|---------|--------|---------|-------|------|--------------------------------|
| [1910'945 | Dec. 11 | 271°82 | 236°51] | ...   | 12°3 | 2 settings each.               |
| 1911'468  | June 20 | 271°79 | 236°43  | ...   | ...  |                                |
| '525      | July 11 | 271°77 | 236°83  | ...   | 12°3 | { Seeing exces-<br>sively bad. |
| '577      | 30      | 271°67 | 236°57  | ...   | 12°0 |                                |
| 1911'523  |         | 271°74 | 236°61  | ...   | 12°1 |                                |

## 30 and 3.

|           |         |        |         | Mags. |      |                  |
|-----------|---------|--------|---------|-------|------|------------------|
| [1910'945 | Dec. 11 | 278°92 | 217°81] | ...   | ...  | 2 settings each. |
| 1911'577  | July 30 | 278°76 | 217°97  | ...   | 13°7 |                  |
| '582      | Aug. 1  | 278°89 | 217°67  | ...   | ...  |                  |
| '591      | 4       | 278°69 | 217°78  | ...   | ...  |                  |
| 1911'583  |         | 278°78 | 317°81  | ...   | 13°7 |                  |

## 30 and 6.

|           |         |        |         | Mags. |      |                  |
|-----------|---------|--------|---------|-------|------|------------------|
| [1910'912 | Nov. 29 | 254°60 | 163°16] | ...   | 12   | 1 setting each.  |
| '931      | Dec. 6  | 254°67 | 163°15  | ...   | 13°2 | 3 settings each. |
| 1911'213  | Mar. 19 | 254°66 | 163°23  | ...   | 13   |                  |
| '270      | Apr. 9  | 254°66 | 163°68  | ...   | ...  |                  |
| '290      | 16      | 254°63 | 163°33  | ...   | ...  |                  |
| '309      | 23      | 254°71 | 163°24  | ...   | ...  |                  |
| 1911'203  |         | 254°67 | 163°33  | ...   | 12°7 |                  |

## 30 and 7.

|           |        |        |         | Mags. |      |                                |
|-----------|--------|--------|---------|-------|------|--------------------------------|
| [1910 931 | Dec. 6 | 303°10 | 170°09] | ...   | ...  | 1 setting each.                |
| '945      | 11     | 302°32 | 171°10] | ...   | ...  | 2 settings each.               |
| 1911'500  | July 2 | 302°86 | 170°32  | ...   | 12°9 |                                |
| '525      | 11     | 303°05 | 170°61  | ...   | 13   | { Seeing exces-<br>sively bad. |
| '577      | 30     | 302°90 | 170°36  | ...   | 12°7 |                                |
| 1911'534  |        | 302°94 | 170°43  | ...   | 12°9 |                                |

## 30 and 8.

|           |         |        |         | Mags. |      |                                |
|-----------|---------|--------|---------|-------|------|--------------------------------|
| [1910'945 | Dec. 11 | 292°97 | 144°06] | ...   | ...  | 2 settings each.               |
| 1911'462  | June 18 | 293°47 | 144°54  | ...   | ...  |                                |
| '500      | July 2  | 293°03 | 144°48  | ...   | 13°2 |                                |
| '525      | 11      | 293°37 | 144°85  | ...   | 13   | { Seeing exces-<br>sively bad. |
| '539      | 16      | 293°24 | 144°35  | ...   | ...  |                                |
| 1911'506  |         | 293°28 | 144°56  | ...   | 13°1 |                                |

30 and 9.

|          |         |        |         | Mags. |      |                     |
|----------|---------|--------|---------|-------|------|---------------------|
|          |         |        |         | ...   | ...  |                     |
| 1911'468 | June 20 | 210°02 | 250''21 | ...   | ...  | V. faint in clouds. |
| '582     | Aug. 1  | 210°05 | 250°70  | ...   | 12°5 |                     |
| '591     | 4       | 210°06 | 250°38  | ...   |      |                     |
| '596     | 6       | 209°87 | 250°83  | ...   |      |                     |
| <hr/>    |         | <hr/>  |         | <hr/> |      |                     |
| 1911'559 |         | 210°00 | 250°53  | ...   | 12°5 |                     |

30 and 10.

|           |         |        |          | Mags. |      |                  |
|-----------|---------|--------|----------|-------|------|------------------|
|           |         |        |          | ...   | ...  |                  |
| [1910'945 | Dec. 11 | 312°02 | 168''73] | ...   | ...  | 2 settings each. |
| 1911'500  | July 2  | 311°97 | 168°16   | ...   | 13°3 |                  |
| '525      | 11      | 312°21 | 168°46   | ...   | 13°4 | { Seeing exces-  |
| '577      | 30      | 312°22 | 168°44   | ...   | 13°7 |                  |
| <hr/>     |         | <hr/>  |          | <hr/> |      | sively bad.      |
| 1911'534  |         | 312°13 | 168°26   | ...   | 13°5 |                  |

30 and 11.

|           |         |        |          | Mags. |      |                 |
|-----------|---------|--------|----------|-------|------|-----------------|
|           |         |        |          | ...   | ...  |                 |
| [1910'912 | Nov. 29 | 275°30 | 120''00] | ...   | ...  | 1 setting each. |
| 1911'468  | June 20 | 275°54 | 120°72   | ...   |      |                 |
| '500      | July 2  | 275°57 | 120°62   | ...   | 13°3 | { Seeing exces- |
| '525      | 11      | 275°44 | 120°52   | ...   | 13   |                 |
| '539      | 16      | 275°29 | 120°30   | ...   |      |                 |
| '582      | Aug. 1  | 275°80 | 120°53   | ...   |      |                 |
| <hr/>     |         | <hr/>  |          | <hr/> |      |                 |
| 1911'523  |         | 275°53 | 120°54   | ...   | 13°2 |                 |

30 and 12.

|           |         |        |          | Mags. |       | 2 settings each. |
|-----------|---------|--------|----------|-------|-------|------------------|
|           |         |        |          | ...   | ...   |                  |
| [1910'945 | Dec. 11 | 267°42 | 116''10] | ...   | ...   |                  |
| 1911'500  | July 2  | 267°08 | 116°24   | ...   | 14'5  |                  |
| '591      | Aug. 4  | 267°27 | 116°40   |       |       |                  |
| '615      | 13      | 266°94 | 116°18   |       |       |                  |
| <hr/>     |         | <hr/>  |          | <hr/> | <hr/> |                  |
| 1911'569  |         | 267°10 | 115°31   | ...   | 14'5  |                  |

30 and 14.

|           |         |        |          | Mags. |     |                                  |
|-----------|---------|--------|----------|-------|-----|----------------------------------|
| [1910'945 | Dec. 11 | 225°32 | 159''87] | ...   | ... | 2 settings each.                 |
| 1911'468  | June 20 | 223°45 | 159°40   | ...   | ... | { Very faint in<br>dense clouds. |
| '506      | July 4  | 223°30 | 159°31   |       |     |                                  |
| '525      | 11      | 223°31 | 159°46   | ...   | ... | { Seeing exces-<br>sively bad.   |
| '539      | 16      | 223°19 | 159°23   |       |     |                                  |
| <hr/>     |         | <hr/>  |          |       |     |                                  |
| 1911'510  |         | 223°31 | 159°35   |       |     |                                  |

30 and 15.

|           |             |        |         | Mags. |      |                                   |
|-----------|-------------|--------|---------|-------|------|-----------------------------------|
|           |             |        |         | ...   | ...  |                                   |
| [1910'945 | Dec. 11     | 245°32 | 103°32] | ...   | ...  | { Very faint.<br>2 settings each. |
| 1911'506  | July 4      | 245°58 | 103°40  | ...   | ...  |                                   |
|           | '577 30     | 245°49 | 103°57  | ...   | 14°2 |                                   |
|           | '582 Aug. 1 | 245°76 | 103°05  | ...   | ...  |                                   |
| 1911'555  |             | 245°61 | 103°34  | ...   | 14°2 |                                   |

30 and 16.

|           |         |        |         | Mags. |     |                                           |
|-----------|---------|--------|---------|-------|-----|-------------------------------------------|
|           |         |        |         | ...   | ... |                                           |
| [1910'912 | Nov. 29 | 241°10 | 100°42] | ...   | ... | 1 setting each.                           |
| 1911'506  | July 4  | 241°57 | 101°22  | ...   | ... |                                           |
|           | '525 11 | 241°53 | 100°91  | ...   | ... | { Seeing exces-<br>sively bad.            |
|           | '558 23 | 241°33 | 101°01  | ...   | ... |                                           |
|           | '563 25 | 241°03 | 101°22  | ...   | ... | { A lesser star<br>5"-10" south<br>of 16. |
| 1911'538  |         | 241°36 | 101°09  | ...   | ... |                                           |

30 and 17.

|           |         |        |        | Mags. |      |                  |
|-----------|---------|--------|--------|-------|------|------------------|
|           |         |        |        | ...   | ...  |                  |
| [1910'950 | Dec. 13 | 288°32 | 89°54] | ...   | ...  | 2 settings each. |
| 1911'500  | July 2  | 288°59 | 89°79  | ...   | 13°8 |                  |
|           | '563 25 | 288°80 | 90°03  | ...   | ...  |                  |
|           | '577 30 | 289°14 | 90°41  | ...   | 14°7 |                  |
| 1911'547  |         | 288°84 | 90°08  | ...   | 14°2 |                  |

30 and 18.

|           |             |        |         | Mags. |      |                                |
|-----------|-------------|--------|---------|-------|------|--------------------------------|
|           |             |        |         | ...   | ...  |                                |
| [1910'950 | Dec. 13     | 328°75 | 158°84] | ...   | ...  | 2 settings each.               |
| 1911'468  | June 20     | 328°65 | 158°34  | ...   | ...  |                                |
|           | '500 July 2 | 328°48 | 158°57  | ...   | 12°9 |                                |
|           | '525 11     | 328°63 | 158°70  | ...   | 13   | { Seeing exces-<br>sively bad. |
| 1911'498  |             | 328°59 | 158°54  | ...   | 12°9 |                                |

30 and 19.

|          |         |        |         | Mags. |     |                                                       |
|----------|---------|--------|---------|-------|-----|-------------------------------------------------------|
|          |         |        |         | ...   | ... |                                                       |
| 1910'912 | Nov. 29 | 229°30 | 105°09] | ...   | ... | 1 setting each.                                       |
| 1911'506 | July 4  | 229°22 | 105°67  | ...   | ... |                                                       |
|          | '525 11 | 229°01 | 105°35  | ...   | ... | { Seeing exces-<br>sively bad.<br>V. faint in clouds. |
|          | '544 18 | 229°49 | 105°50  | ...   | ... |                                                       |
| 1911'525 |         | 229°24 | 105°51  | ...   | ... |                                                       |

30 and 22.

|           |         |                                    |         | Mags.                     |     |                             |
|-----------|---------|------------------------------------|---------|---------------------------|-----|-----------------------------|
| [1910'912 | Nov. 29 | 250°70                             | 69''71] | ...                       | ... | 1 setting each.             |
| 1911'500  | July 2  | 250°60                             | 69°28   | ...                       | ... |                             |
| '591      | Aug. 4  | 250°55                             | 69°22   | ...                       | ... |                             |
| '615      | 13      | 250°10                             | 68°91   | ...                       | ... | { Very difficult in clouds. |
| <hr/>     |         | <hr/>                              |         | <hr/>                     |     |                             |
| 1911'569  |         | 250°42                             | 69°14   | ...                       | ... |                             |
|           |         | This gives $\Delta\alpha - 5^s.43$ |         | $\Delta\delta - 23''.2$ . |     |                             |

30 and 23.

|           |         |        |         | Mags. |     |                 |
|-----------|---------|--------|---------|-------|-----|-----------------|
| [1910'912 | Nov. 29 | 263°80 | 62''07] | ...   | ... | 1 setting each. |
| 1911'500  | July 2  | 263°38 | 61°15   | ...   | 15½ |                 |
| '621      | Aug. 15 | 263°01 | 60°97   | ...   | 15½ | Very difficult. |
| <hr/>     |         | <hr/>  |         | <hr/> |     |                 |
| 1911'560  |         | 263°20 | 61°06   | ...   | 15½ |                 |

30 and 24.

|           |         |        |          | Mags. |     |                           |
|-----------|---------|--------|----------|-------|-----|---------------------------|
| [1910'950 | Dec. 13 | 194°77 | 152''73] | ...   | ... | 2 settings each.          |
| 1911'468  | June 20 | 194°67 | 151°84   | ...   | 14  |                           |
| '525      | July 11 | 194°82 | 151°68   | ...   | ... | { Seeing excessively bad. |
| '539      | 16      | 194°76 | 151°98   | ...   | ... |                           |
| <hr/>     |         | <hr/>  |          | <hr/> |     |                           |
| 1911'511  |         | 194°75 | 151°83   | ...   | 14  |                           |

30 and 25.

|           |         |        |         | Mags. |      |                                     |
|-----------|---------|--------|---------|-------|------|-------------------------------------|
| [1910'912 | Nov. 29 | 316°30 | 47''46] | ...   | ...  | 1 setting each.                     |
| 1911'314  | Apr. 25 | 317°81 | 48°20   | ...   | 13°8 |                                     |
| '333      | May 2   | 317°46 | 47°73   | ...   | 14°2 |                                     |
| '391      | 23      | 317°64 | 47°71   | ...   | 13°0 |                                     |
| '404      | 28      | 317°24 | 48°12   | ...   | ..   | Faint in clouds.                    |
| '424      | June 4  | 317°34 | 47°74   | ...   | ...  |                                     |
| '495      | 30      | 317°77 | 47°17   | ...   | ...  |                                     |
| '577      | July 30 | 317°59 | 47°63   | ...   | 14°2 |                                     |
| '582      | Aug. 1  | 318°06 | 47°02   | ...   | ...  | { Very faint and difficult in haze. |
| <hr/>     |         | <hr/>  |         | <hr/> |      |                                     |
| 1911'440  |         | 317°61 | 47°66   | ..    | 13°8 |                                     |

30 and 26.

|           |         |        |         | Mags. |      |                   |
|-----------|---------|--------|---------|-------|------|-------------------|
| [1910'912 | Nov. 29 | 242°90 | 25''23] | ...   | ...  | 1 setting each.   |
| 1911'314  | Apr. 25 | 241°87 | 24°36   | ...   | 14   |                   |
| '333      | May 2   | 242°28 | 24°82   | ...   | 13°8 |                   |
| '372      | 16      | 241°13 | 24°79   | ...   | ...  | V. faint in haze. |
| '391      | 23      | 241°82 | 24°90   | ...   | 13   |                   |
| '405      | 28      | 241°96 | 24°36   | ...   | ...  | In clouds.        |
| <hr/>     |         | <hr/>  |         | <hr/> |      |                   |
| 1911'363  |         | 241°81 | 24°65   | ...   | 13°6 |                   |

26 and 27.

|          |         |       |       | Mags. |      |                                                |
|----------|---------|-------|-------|-------|------|------------------------------------------------|
|          |         |       |       |       |      |                                                |
| 1912'426 | June 4  | 97°84 | 8''06 | ...   | 14'5 | { Very difficult,<br>faint from bad<br>seeing. |
| '459     | 16      | 97°29 | 7'94  | ...   | 13'5 |                                                |
| '475     | 22      | 98°85 | 7'81  | ...   | 15   |                                                |
| '532     | July 13 | 99°95 | 7'74  | ...   | ...  |                                                |
| 1912'473 |         | 98°48 | 7'89  | ...   | 14'3 |                                                |

30 and 28.

|          |        |        |       | Mags. |    |                                |
|----------|--------|--------|-------|-------|----|--------------------------------|
|          |        |        |       |       |    |                                |
| 1912'426 | June 4 | 263°28 | 6''74 | ...   | 15 | { Very faint and<br>difficult. |
| '752     | Oct. 1 | 260°69 | 6'63  | ...   | 15 |                                |
| 1912'589 |        | 261°98 | 6'68  | ...   | 15 |                                |

30 and 29.

|          |         |        |        | Mags. |      |                                         |
|----------|---------|--------|--------|-------|------|-----------------------------------------|
|          |         |        |        |       |      |                                         |
| 1912'426 | June 4  | 353°69 | 28''80 | ...   | 13'6 | Faint in clouds.                        |
| '434     | 7       | 353°58 | 28'99  | ...   | ...  |                                         |
| '459     | 16      | 354°89 | 29'11  | ...   | 13'3 | { A 14½ <sup>m</sup> star 3''<br>± s.p. |
| '475     | 22      | 354°97 | 28'89  | ...   | ...  |                                         |
| '532     | July 13 | 354°80 | 28'57  | ...   | ...  | { Very faint in<br>thick sky.           |
| 1912'465 |         | 354°39 | 28'87  | ...   | 13'4 |                                         |

30 and 31.

|           |         |       |         | Mags. |     |                                 |
|-----------|---------|-------|---------|-------|-----|---------------------------------|
|           |         |       |         |       |     |                                 |
| [1910'950 | Dec. 13 | 14°96 | 50''75] | ...   | ... | { 2 angle, 3 dist.<br>settings. |
| 1911'468  | June 20 | 14°03 | 51'19   | ...   | ... |                                 |
| '487      | 27      | 14°68 | 50'88   | ...   | ... |                                 |
| '506      | July 4  | 14°58 | 50'84   | ...   | ... |                                 |
| '539      | 16      | 14°57 | 51'14   | ...   | ... |                                 |
| 1911'500  |         | 14°46 | 51'01   | ...   | ... |                                 |

30 and 32.

|           |         |       |         | Mags. |     |                  |
|-----------|---------|-------|---------|-------|-----|------------------|
|           |         |       |         |       |     |                  |
| [1910'950 | Dec. 13 | 44°17 | 59''68] | ...   | ... | 2 settings each. |
| 1911'468  | June 20 | 44°00 | 59'45   | ...   | ... |                  |
| '487      | 27      | 44°10 | 60'08   | ...   | ... |                  |
| '495      | 30      | 44°22 | 59'23   | ...   | ... |                  |
| 1911'483  |         | 44°11 | 59'59   | ...   | ... |                  |

| 30 and 33. |             |        |         | Mags. |     | I setting each. |
|------------|-------------|--------|---------|-------|-----|-----------------|
|            |             |        |         | ...   | ... |                 |
| [1910'912  | Nov. 29     | 139°00 | 112°48] |       |     |                 |
| 1911'213   | Mar. 19     | 139°26 | 111°95  |       |     |                 |
|            | '232 26     | 139°14 | 112°24  |       |     |                 |
|            | '270 Apr. 9 | 138°96 | 112°36  |       |     |                 |
|            | '290 16     | 139°07 | 112°41  |       |     |                 |
| <hr/>      |             | <hr/>  |         |       |     |                 |
| 1911'251   |             | 139°11 | 112°24  |       |     |                 |

| 30 and 34. |             |        |         | Mags. |     | I setting each. |
|------------|-------------|--------|---------|-------|-----|-----------------|
|            |             |        |         | ...   | ... |                 |
| [1910'912  | Nov. 29     | 102°80 | 113°31] |       |     |                 |
| 1911'213   | Mar. 19     | 103°02 | 113°30  |       |     |                 |
|            | '232 26     | 102°72 | 113°05  |       |     |                 |
|            | '270 Apr. 9 | 102°83 | 113°25  |       |     |                 |
|            | '290 16     | 102°88 | 113°29  |       |     |                 |
| <hr/>      |             | <hr/>  |         |       |     |                 |
| 1911'251   |             | 102°86 | 113°22  |       |     |                 |

From these measures the accompanying chart of this region has been prepared.

Following are estimates of the magnitudes of the stars, including those given with the measures:—

| Star. | Mag. | No. Ests. | Star.    | Mag.                   | No. Ests. |
|-------|------|-----------|----------|------------------------|-----------|
| 1     | 12'3 | 3         | 18       | 13'0                   | 2         |
| 2     | 12'3 | 5         | 19       | 13'6                   | 4         |
| 3     | 13'7 | 1         | 20       | 16                     | 1         |
| 4     | 11'1 | 3         | 21       | 16                     | 1         |
| 5     | 13'6 | 2         | (20, 21) | 15'5                   | 2         |
| 6     | 12'7 | 5         | 22       | 14'5                   | 4         |
| 7     | 12'9 | 3         | 23       | 15'4                   | 4         |
| 8     | 13'1 | 2         | 24       | 14                     | 1         |
| 9     | 12'5 | 1         | 25       | 13'8                   | 4         |
| 10    | 13'5 | 3         | 26       | 13'7                   | 4         |
| 11    | 13'3 | 6         | 27       | 14'3                   | 3         |
| 12    | 14'2 | 4         | 28       | 15'0                   | 2         |
| 13    | 13'8 | 2         | 29       | 14'3                   | 2         |
| 14    | 13'6 | 2         | 30       | 11'5                   | 4         |
| 15    | 14'2 | 5         | 31       | 13'9 (Est. from photo) |           |
| 16    | 13'9 | 4         | 32       | 13                     | „         |
| 17    | 14'2 | 2         | 33       | 11'3                   | 1         |
|       |      |           | 34       | 11'3                   | 1         |

Estimates of the relative light of some of the stars were made

at various times. They are given here in the hope that they may some time be useful.

1910 Dec. 11. Star 22 was the same brightness as star 15. Seeing very bad; stars masses of woolly light. Wind shaking telescope.

Dec. 13. Star 22 was the mean brightness of stars 15 and 16.

1911 March 19. Star 6 was  $0^m.1$  less than star 30. Star 30 was equal to star 33, and was  $0^m.3$  brighter than star 34.

March 26. Star 34 was  $0^m.2$  brighter than star 33.

April 25. Star 12 was  $1\frac{1}{2}^m$  less than star 11.

May 2. Stars 2 and 6 were of equal magnitude; possibly 2 was slightly brighter, but the seeing was too bad to tell certainly. Star 16 was  $0^m.1$  brighter than star 15.

May 16. Star 5 was  $0^m.3$  less than star 14, and star 2 was  $0^m.1$  less than star 6.

May 23. Star 2 was  $0^m.4$  brighter than star 6.

May 28. Stars 2 and 6 were of equal magnitude.

July 30. Star 15 was slightly less than star 16.

Aug. 13. Stars 12 and 22 were of equal brightness.

1912. Nov. 15. Star 34 was  $0^m.3$  brighter than star 33.

#### *Notes on the Brightness of the Variable.*

It may be well to state that direct estimates of the magnitude of a star with a large telescope are subject to much uncertainty and discordance under different conditions of seeing, etc., much more so, perhaps, than with a small telescope. The estimates of the magnitude of the variable should have much less value than a comparison of its light with that of the other stars, when the magnitudes of the comparison stars have been determined.

1910.

Nov. 6,  $9^h 0^m$ . There were two or three very faint stars within  $5'' \pm$  of star 22. Observations through breaks in clouds.

Nov. 13,  $7^h 40^m$ . Could faintly see star 22 near the place of Hinks's variable. Adjusted the wires, with the aid of star 22, to a point  $12''$  preceding 22, and the same declination. There was nothing seen near this point that would be as bright as the 16th magnitude. Clouds stopped the observations.

Nov. 24,  $7^h 0^m$ . Identified the variable by the following measures of it with respect to star 30,

$$\Delta\alpha \cos \delta - 80''.6 (4) = \Delta\alpha - 68''.71 \quad \Delta\delta - 27''.0 (3).$$

These, though somewhat discordant on account of the faintness of the star, proved the object seen was the variable. It was excessively faint. Estimated magnitude,  $16\frac{3}{4}$ . With a power of 700 it was a little better seen, and was estimated to be  $16\frac{1}{2}$  magnitude.

1911.

March 19,  $15^h 40^m$ . The variable was  $0^m.3$  brighter than star 30,  $1^m$  brighter than star 6, and  $0^m.2$  less than star 2.

Compared with 30 it was slightly yellowish. There was nothing peculiar in its out-of-focus image. Focal measures were made of the variable and star 30.

Variable, 6. mm. (3).

Star 30, 5. mm. (3).

Seeing very bad and focus difficult. The difference is too small to mean anything.

March 26, 15<sup>h</sup> 0<sup>m</sup>. It seemed to be the same magnitude as star 30, possibly 0<sup>m</sup>.1 brighter.

April 9, 15<sup>h</sup> 0<sup>m</sup>. It was 0<sup>m</sup>.4 brighter than star 30.

April 16, 14<sup>h</sup> 45<sup>m</sup>. It was 0<sup>m</sup>.1 brighter than star 30, and was estimated to be 10.8 magnitude.

April 23, 14<sup>h</sup> 45<sup>m</sup>. Star 30 and the variable were almost exactly alike. Perhaps the variable was 0<sup>m</sup>.05 less than 30. It was slightly yellowish. It was 0.6 the light from star 6 to star 30. Seeing good.

April 25, 15<sup>h</sup> 20<sup>m</sup>. The variable was from 0<sup>m</sup>.3 to 0<sup>m</sup>.4 less than star 30. It was about the mean brightness of stars 6 and 30.

May 2, 14<sup>h</sup> 20<sup>m</sup>. It was 0<sup>m</sup>.5 less than star 30, and 0<sup>m</sup>.1 brighter than star 6. Seeing bad.

May 7, 15<sup>h</sup> 10<sup>m</sup>. The same brightness as star 6 and 1<sup>m</sup> less than star 30. It was the same as star 33. Seen only through passing breaks in clouds.

May 16, 14<sup>h</sup> 0<sup>m</sup>. The variable was 0<sup>m</sup>.1 less than star 8, 0<sup>m</sup>.2 less than star 6, and 2<sup>m</sup> less than 30. It was 0.4 brighter than star 11, and 0.4 less than star 3.

May 23, 13<sup>h</sup> 50<sup>m</sup>. It was the same brightness as star 11, and 0<sup>m</sup>.4 brighter than star 14.

May 28, 14<sup>h</sup> 0<sup>m</sup>. The variable was the same brightness as star 14.

June 4, 14<sup>h</sup> 0<sup>m</sup>. The same brightness as star 14, or perhaps a little fainter. It was 0<sup>m</sup>.2 or 0<sup>m</sup>.3 brighter than star 16.

June 18, 14<sup>h</sup> 0<sup>m</sup>. It was equal in brightness to star 16,  $\frac{1}{2}$ <sup>m</sup> brighter than star 22, and was estimated to be 14th magnitude.

June 20, 13<sup>h</sup> 30<sup>m</sup>. It was the same brightness as star 16, and was 0<sup>m</sup>.4 less than star 19.

June 27, 14<sup>h</sup> 10<sup>m</sup>. The variable was 0<sup>m</sup>.4 less than star 22. It was 1<sup>m</sup> less than star 15. Estimated magnitude, 14.7.

June 30, 13<sup>h</sup> 50<sup>m</sup>. It was 0<sup>m</sup>.6 less than star 22. It was less than star 15, and was between stars (20, 21) and 22 in brightness. Estimated magnitude, 15 or 15 $\frac{1}{2}$ .

July 2, 13<sup>h</sup> 30<sup>m</sup>. It was 0<sup>m</sup>.5 less than star 15, 0<sup>m</sup>.7 less than star 22, and was the same brightness as star 23.

July 4, 11<sup>h</sup> 0<sup>m</sup>. The variable was 1<sup>m</sup> or 1 $\frac{1}{2}$ <sup>m</sup> less than star 22 and was 0<sup>m</sup>.1 brighter than star 23. It was faint in moonlight—about 15 $\frac{1}{2}$  or 16th magnitude.

July 11, 10<sup>h</sup> 45<sup>m</sup>. Equal to star 23, and 1 $\frac{1}{2}$ <sup>m</sup> to 2<sup>m</sup> fainter than star 22. Seeing very poor. Full moon. Variable very faint.

July 16, 14<sup>h</sup> 10<sup>m</sup>. The variable was the same brightness as star 23. It was very faint. Bright moon.

July 23, 15<sup>h</sup> 15<sup>m</sup>. Very faint—16th magnitude or fainter. It was 1<sup>m</sup> fainter than star (20, 21).

July 25, 13<sup>h</sup> 50<sup>m</sup>. About 15.7 magnitude. It was 1½<sup>m</sup> less than star 23. There are two stars of the same magnitude (15<sup>m</sup>) at the place of star (20, 21). No faint star very close to the variable.

July 30, 8<sup>h</sup> 20<sup>m</sup>. The variable was 16½ magnitude. It was very faint. It was 1<sup>m</sup> fainter than star 24. Seeing good, sky good.

Aug. 1, 11<sup>h</sup> 20<sup>m</sup>. Variable occasionally glimpsed. Same brightness as on July 30.

Aug. 4, 11<sup>h</sup> 30<sup>m</sup>. Very faint. It was 1<sup>m</sup> or 1½<sup>m</sup> fainter than star (20, 21).

Aug. 15, 11<sup>h</sup> 0<sup>m</sup>. Difficult, 16½ magnitude; very faint. It was 1½<sup>m</sup> or more fainter than the star (20, 21). Seeing good and sky good, but gibbous moon. With 700 diameters.

Aug. 20, 13<sup>h</sup> 15<sup>m</sup>. Very faint, with power 700. Did not see it with 460 diameters. It was at least 16.5 magnitude, and was 2<sup>m</sup> or more fainter than star (20, 21). It was fainter than at the last observation. Seeing poor.

Aug. 22, 11<sup>h</sup> 45<sup>m</sup>. Could not see the variable. The sky was too thick. Could feebly glimpse star 22.

Aug. 29, 11<sup>h</sup> 20<sup>m</sup>. Could not see the variable with 460 diameters. Seeing fair. With 700 diameters it must have been excessively faint—at least 2<sup>m</sup> less than star (20, 21). Could not be sure of seeing it. The seeing was not good, but the variable must have been very much fainter than at the last observation of it.

Sept. 1, 11<sup>h</sup> 30<sup>m</sup>. Could not see it with 460 diameters. Sky fair, seeing fair. By waiting and watching with 700 diameters could occasionally catch feeble glimpses of the star. It was excessively faint, and must have been between 16½ and 17 magnitude. Seeing by moments was quite good, but very high wind.

Sept. 3, 7<sup>h</sup> 45<sup>m</sup>. Gibbous moon. Sky too poor to look for the variable. Seeing poor. Could just see star 23.

Sept. 19, 9<sup>h</sup> 40<sup>m</sup>. Thought I could once in a while feebly see it. There was possibly a faint star slightly farther away from star 23. If so, the variable was much fainter than this star. It could not be brighter than the limit of the 40-inch. Seeing good, sky good. 700 diameters.

Sept. 26, 9<sup>h</sup> 30<sup>m</sup>. Seeing poor, sky not good. Stars 22 and 23 and the two stars 20 and 21 seen, but could see nothing of the variable. It must have been at least 1<sup>m</sup> less than either 20 or 21. With 700 and 460 diameters.

Oct. 3, 10<sup>h</sup> 10<sup>m</sup>. Could not see the star. Could see star 22 and feebly glimpse star 23. The variable must have been less than either of these stars. The stars were faint, perhaps from moisture on the object-glass.

Oct. 20, 8<sup>h</sup> 40<sup>m</sup>. Could see the stars 22, 23, and the two stars 20 and 21. Could not be certain of seeing the variable, but thought I glimpsed it. It must have been  $\frac{1}{2}^m$  less than the two stars 20 and 21, if seen at all. With 700 diameters.

Oct. 24, 6<sup>h</sup> 45<sup>m</sup>. Could see it quite steadily occasionally. It was perhaps  $\frac{1}{2}^m$  less than either star 20 or 21. Identified the variable with certainty by measures with respect to star 22.

P.A. 258°·4 (3). Dist. 14"·05 (4 single distances).

Oct. 27, 7<sup>h</sup> 40<sup>m</sup>. Could see it fairly well. It was about the same brightness as the two stars 20 and 21 (which are 3" ± apart east and west). Star 21 is slightly the brighter. Star 20 is about the same brightness as the variable.

Oct. 29, 6<sup>h</sup> 0<sup>m</sup>. The variable was seen steadily. Sky good, seeing fair. It was about 1<sup>m</sup> less than star 23. It was slightly less than the fainter of the two stars 20 and 21. Fairly easy, but faint. With 700 diameters. There was a 15th magnitude star 15" or 20" south following star 3.

#### 1912.

May 17, 13<sup>h</sup> 35<sup>m</sup>. The variable was very faint; could only glimpse it. It was fainter than stars (20, 21) or 23, which I could see faintly at odd times and which were fainter than star 22. Sky thick and seeing very bad.

June 4, 12<sup>h</sup> 35<sup>m</sup>. Could see it very feebly, about 1<sup>m</sup> less than star (20, 21), and very much less than star 23. Could just see it and no more. Seeing very poor, 700 diameters. The place of star (20, 21) was occupied by two 15½ magnitude stars, 3½" apart. The star supposed to be the variable was 16½ magnitude; certainly not brighter than 16 magnitude. Later: was not certain but there might be more than one star at the place of the variable. Seeing better, but the variable was seen only once in a while.

June 7, 12<sup>h</sup> 10<sup>m</sup>. Very faint. Possibly not so faint as at the observation of June 4. It was 1<sup>m</sup> less than the two stars 20 and 21.

June 11, 12<sup>h</sup> 30<sup>m</sup>. Glimpses of the variable were seen between clouds. Seeing good and sky good for moments. It was very faint, and could not be brighter than 16½ magnitude. The two stars 20 and 21 were easily seen. With 700 diameters.

June 16, 13<sup>h</sup> 45<sup>m</sup>. It was 1<sup>m</sup> less than either star 20 or 21. Sky good. Power 700. Estimated magnitude, 16.

June 22, 12<sup>h</sup> 35<sup>m</sup>. Excessively faint. Could not be brighter than 16½ magnitude. Could just see it by watching closely. It was 1<sup>m</sup> to 1½<sup>m</sup> less than star (20, 21). Sky fair, seeing good. With 700 diameters.

June 25, 9<sup>h</sup> 50<sup>m</sup>. Could not see the variable. Could see 22 and glimpse 23, so it was fainter than either of these stars. Thick sky and nearly full moon. With 700 diameters.

July 2, 11<sup>h</sup> 45<sup>m</sup>. Could not see it. Sky very thick and

moonlit. Could see star 22 and star 23 very feebly. It must have been less than 23. The sky was too thick to see star (20, 21). With 700 diameters.

July 13, 13<sup>h</sup> 0<sup>m</sup>. It was 16th magnitude. Could see it distinctly for moments. It was 1<sup>m</sup> less than either of the two stars 20 or 21. There was another star a little farther from stars (20, 21) and 22. With 700 diameters. Sky good, seeing good. Aperture 30 inches.

July 14, 11<sup>h</sup> 0<sup>m</sup>. It was about 1 or 1½ magnitude less than either of the stars 20 or 21. It was 16½ magnitude. Not seen steadily. Seeing fair, sky good. There was a smaller star a little farther from star 22, perhaps on the line "variable to 22."

Aug. 3, 10<sup>h</sup> 20<sup>m</sup>. Could see it faintly, though the seeing was poor and the sky not very transparent. It was brighter than at the last previous observation. Perhaps ¾<sup>m</sup> less than star (20, 21). With 700 diameters.

Aug. 6, 10<sup>h</sup> 0<sup>m</sup>. The variable was perhaps seen feebly, but the sky was too thick. Could feebly glimpse star (20, 21). Star 22 was fairly well seen. The variable must have been somewhat fainter than star (20, 21). With 700 diameters.

Aug. 11, 11<sup>h</sup> 0<sup>m</sup>. It was ¼<sup>m</sup> brighter than either star 20 or 21 and 1<sup>m</sup> less than star 23. Seeing good, sky good. With 700 diameters.

Aug. 13, 12<sup>h</sup> 20<sup>m</sup>. The star seemed to be slowly brightening. It was ½<sup>m</sup> brighter than either star 20 or 21, and ¾<sup>m</sup> less than star 23.

Aug. 17, 11<sup>h</sup> 20<sup>m</sup>. The star was decidedly brighter. It was the same magnitude as star 23, or possibly 0<sup>m</sup>.1 brighter. It was much less than star 22. With 700 diameters. Through breaks in clouds.

Aug. 20, 8<sup>h</sup> 55<sup>m</sup>. The variable was 0<sup>m</sup>.4 brighter than star 23, and perhaps 0.3 of the brightness from stars 23 to 22. Sky thickish, with ½ moon.

Aug. 24, 10<sup>h</sup> 20<sup>m</sup>. Thick sky, full moon and broken clouds. The variable could be seen once in a while. Could very feebly glimpse star 23. The variable was less than star 22, and brighter than 23. It was perhaps equal to the mean brightness of the two, but uncertain because of the conditions. With 700 diameters.

Aug. 25, 13<sup>h</sup> 40<sup>m</sup>. It was very faint in moonlight. Perhaps 1<sup>m</sup> less than star 22. Could not see 23, so the variable must have been brighter than that star.

Aug. 31, 10<sup>h</sup> 30<sup>m</sup>. It was much brighter. It was about midway in brightness between stars 22 and 16, from ½<sup>m</sup> to ¾<sup>m</sup> brighter than 22, and 0<sup>m</sup>.3 less than 16. With 700 diameters.

Sept. 1, 11<sup>h</sup> 40<sup>m</sup>. The variable was the mean brightness of stars 22 and 16—about ½<sup>m</sup> brighter than 22. It was perhaps 0<sup>m</sup>.2 brighter than star 12. With 700 diameters.

Sept. 7, 10<sup>h</sup> 40<sup>m</sup>. It was the same brightness as star 26, or perhaps 0<sup>m</sup>.1 brighter. It was 1¼<sup>m</sup> brighter than star 22, and perhaps 0<sup>m</sup>.1 or 0<sup>m</sup>.2 less than star 19.

Sept. 8, 11<sup>h</sup> 40<sup>m</sup>. The star was 0<sup>m</sup>·1 brighter than star 26, 0<sup>m</sup>·4 less than star 19, and 1½<sup>m</sup> brighter than 22.

Sept. 10, 10<sup>h</sup> 20<sup>m</sup>. It was the same brightness as star 19 and also as star 26. Faint in thick sky. Seeing poor.

Sept. 17, 9<sup>h</sup> 0<sup>m</sup>. The variable was midway in brightness between stars 11 and 19. Estimated brightness was 12·7 magnitude. Star 23 was very faint.

Sept. 21, 10<sup>h</sup> 40<sup>m</sup>. It was slightly less than star 11. Perhaps ½<sup>m</sup> brighter than star 19 and ½<sup>m</sup> less than star 11, about the mean brightness of the two stars. Seeing very bad. Sky very thick and white from moonlight. With 700 diameters.

Sept. 29, 10<sup>h</sup> 30<sup>m</sup>. It was the mean brightness of stars 8 and 11, and the same brightness as star 19. With 460 diameters.

Oct. 1, 8<sup>h</sup> 0<sup>m</sup>. It was the same brightness as stars 11 and 19. With 700 diameters.

Oct. 5, 9<sup>h</sup> 20<sup>m</sup>. The variable was 0<sup>m</sup>·3 less than star 11, and the same brightness as star 19. Seeing bad. With 700 diameters.

Oct. 12, 8<sup>h</sup> 40<sup>m</sup>. The variable was somewhat brighter, but it brightens slowly. It was 0<sup>m</sup>·5 brighter than star 11, and well defined at moments of steadiness. About 1½<sup>m</sup> less than star 6, and the same brightness as star 7, or perhaps 0<sup>m</sup>·1 or 0<sup>m</sup>·2 fainter than 7. Seeing very poor.

Oct. 15, 8<sup>h</sup> 0<sup>m</sup>. It was ¾<sup>m</sup> brighter than star 11, the same brightness as star 7, and 1<sup>m</sup> less than star 6.

Oct. 22, 8<sup>h</sup> 50<sup>m</sup>. It was almost exactly equal to, or perhaps 0<sup>m</sup>·1 or 0<sup>m</sup>·2 less than star 6.

Oct. 26, 9<sup>h</sup> 50<sup>m</sup>. It was almost, if not exactly, equal to star 6; perhaps 0<sup>m</sup>·1 to 0<sup>m</sup>·2 brighter. Later, better seen: it was 0<sup>m</sup>·2 brighter than 6. It was white.

Oct. 27, 9<sup>h</sup> 35<sup>m</sup>. The variable was 0<sup>m</sup>·3 brighter than star 6, about 0<sup>m</sup>·1 brighter than star 33, and about 0<sup>m</sup>·3 less than 34.

Oct. 29, 7<sup>h</sup> 50<sup>m</sup>. It was 0<sup>m</sup>·2 brighter than star 6—almost the same. It was 0<sup>m</sup>·1 brighter than star 33, and 0<sup>m</sup>·4 less than 34. Perhaps it was the mean magnitude of stars 6 and 34.

Nov. 7, 9<sup>h</sup> 20<sup>m</sup>. It was 0<sup>m</sup>·2 brighter than star 6, and 0<sup>m</sup>·1 or 0<sup>m</sup>·2 brighter than star 34. Later, with 700 diameters: it was 0<sup>m</sup>·3 brighter than star 6, and ¾<sup>m</sup> to 1<sup>m</sup> less than star 30.

Nov. 10, 9<sup>h</sup> 30<sup>m</sup>. The variable was 0<sup>m</sup>·2 brighter than star 6, and 0<sup>m</sup>·1 or 0<sup>m</sup>·2 brighter than star 34. It was apparently colourless and looked like the other stars. Seeing very bad.

Nov. 15, 6<sup>h</sup> 45<sup>m</sup>. It was 0<sup>m</sup>·4 brighter than star 6, 1<sup>m</sup> less than 30, 0<sup>m</sup>·2 brighter than 33, and 0<sup>m</sup>·4 less than star 34. It was probably the mean brightness of 33 and 34. Seeing good.

Dec. 9, 5<sup>h</sup> 15<sup>m</sup>. It was the same brightness as star 6. Later, a little better seen: it was 0<sup>m</sup>·1 brighter than star 6, 0<sup>m</sup>·2 less than 33, and 0<sup>m</sup>·3 brighter than star 5.

Dec. 31, 5<sup>h</sup> 20<sup>m</sup>. The variable had faded greatly since the last observation. It was 0<sup>m</sup>·3 brighter than star 11, and 0<sup>m</sup>·2 less than star 7, or about the mean brightness of these stars. Seeing poor.

1913.

March 13, 16<sup>h</sup> 30<sup>m</sup>. Faintly seen. Estimated 16th magnitude. It was 1<sup>m</sup> or 1½<sup>m</sup> fainter than star 23, and from ½<sup>m</sup> to ¾<sup>m</sup> less than star (20, 21). Sky poor, seeing poor. Magnifying power 700 diameters.

The times throughout this paper are Central Meridian Time, and are 6<sup>h</sup> 0<sup>m</sup> slow of Greenwich Mean Time.

*Yerkes Observatory,  
Williams Bay, Wisconsin,  
1913 March 18.*

*P.S.*—1913 April 14, 16<sup>h</sup> 0<sup>m</sup>. Estimated 16th magnitude. It was 1<sup>m</sup> less than star 23, and ¾<sup>m</sup> less than either star 20 or 21. There was a fainter star close preceding, on the line of variable and 22, which was ½<sup>m</sup> or ¾<sup>m</sup> less than the variable. Power 700. Seeing good.

---

*On the Photographic Magnitudes determined with the Greenwich  
Astrographic Equatorial: Corrections depending on distance  
from the plate-centre.*

*(Communicated by the Astronomer Royal.)*

The present investigation was made in connection with the determination of photographic magnitudes for the *Greenwich Astrographic Catalogue*. It is well known that the telescopes used for the *Carte du Ciel* do not give uniform definition over the whole field used; and accordingly, in deducing the magnitudes from the diameters of star-images, it is desirable to apply a correction depending on the distance from the centre of the plate. This correction is particularly important, since the standard stars of the Harvard sequence fall on a comparatively small area in the centre of the plate, and the deduced magnitudes of the other stars scattered over a larger area may in the mean be 0<sup>m</sup>·1 or 0<sup>m</sup>·2 too bright. A very sensitive means of determining these corrections is afforded by counting the relative numbers of stars photographed in different parts of the plate.

The behaviour of the Oxford, Paris, Algiers, Toulouse, and San Fernando telescopes has been discussed by Professor Turner (*M.N.*, lxii. p. 434, and lxviii. p. 397). It was found that the relative density of the stars on the plates gradually increases, as we proceed from the centre to a certain distance, and then decreases rather rapidly towards the edges. This is due to the fact that these telescopes are so focussed that the best definition is found over an annulus, the centre of the plate being just inside focus, and the edges outside focus. Similar results were found for the Greenwich telescope (*Astrographic Catalogue*, vol. i. p. vii), the zone of maximum density being in this case at 40' from the centre.

These investigations refer to the limiting magnitude attained at different parts of the plate, being based on counts of all the stars shown. The determination of the corresponding effect on the measured diameters of the images presents a slightly different problem. An interesting point is that the imperfect definition at the edges of the plate affects bright and faint stars in opposite ways; the diameter of a bright star is made larger by being slightly out of focus, whereas a faint star may either be lost entirely, or will leave a fainter impression, which will be judged to have a smaller diameter.

In order to determine definite numerical corrections, the distribution of the stars between various limits of measured diameter was studied in detail on a large number of plates. Taking zones  $72^\circ$ ,  $73^\circ$ ,  $75^\circ$ ,  $76^\circ$ ,  $77^\circ$ , and  $79^\circ$  of the *Astrographic Catalogue*, the number of stars in each réseau square was counted:

(1) For the top left quadrant ( $x = 1 - 14$ ,  $y = 14 - 27$ ) of each plate; this was done separately for diameters 4, 5, 6 and 7, 8 to 10, 11 to 14, 15 to 19, 20 to 29, 30 to 49, 50 and over.

(2) For the whole plate; for diameters 30 to 49, 50 and over.

These counts comprise 27,800 stars.

Apart from accidental deviations, the number of stars down to any limit of *magnitude* must be the same in each square. (The increase of area, when a region of the sky is projected on the plane, is not more than 1 in 1000 at any part of the plate.) If, then, the number down to any limit of *diameter* is found not to be the same, the diameter must correspond to different magnitudes according to position on the plate. Making no assumption as to the formula connecting magnitude and diameter, we can determine by interpolation from the counts corrections to the diameters measured in different parts of the plate, which will reduce them to a common standard. The precise procedure will be stated later.

In the following tables the réseau squares have been grouped into twelve annuli (of zigzag outline) according to distance from the centre. The annuli each contain about eleven squares (per quadrant), except the first and last, which have fewer. It is necessary to remember that the whole plate is not measured for the *Astrographic Catalogue*; certain squares at the edges are unmeasured, and of others part only is measured. The boundary of the measured part was drawn for each zone in order to determine the precise fraction of each imperfect square included in the annuli. The "transition zones"  $74^\circ$  and  $78^\circ$  would have presented greater difficulties than the others in making this estimation, and were therefore not used.

Table I. contains the number of stars between successive limits of diameter for each annulus.

In Table II. running totals of the columns of Table I. have been formed, and divided by the number of réseau squares in the annulus. The table exhibits the number of stars per réseau square with diameters greater than successive limits.

TABLE I.—Number of Stars counted in Successive Annuli.

| Annulus              | a      | b       | c       | d       | e            | f       | g       | h       | j       | k       | l       | m       |
|----------------------|--------|---------|---------|---------|--------------|---------|---------|---------|---------|---------|---------|---------|
| Distance from centre | 0'-17' | 17'-25' | 25'-31' | 31'-37' | 37'-41'      | 41'-45' | 45'-49' | 49'-53' | 53'-57' | 57'-61' | 61'-68' | 68'-78' |
| No. of squares       | 8      | 11      | 11      | 11      | 11           | 12      | 11      | 12.7    | 10.6    | 11.5    | 10.8    | 5.3     |
| Diameter > 50        | 27     | 40      | 34      | 25      | 40           | 48      | 45      | 34      | 31      | 36      | 51      | 30      |
| 30-49                | 65     | 94      | 93      | 92      | 103          | 108     | 98      | 109     | 123     | 135     | 156     | 80      |
| 20-29                | 136    | 166     | 190     | 213     | 229          | 296     | 232     | 275     | 243     | 216     | 218     | 81      |
| 15-19                | 156    | 199     | 198     | 222     | 219          | 221     | 230     | 217     | 168     | 187     | 167     | 76      |
| 11-14                | 150    | 217     | 221     | 194     | 217          | 242     | 217     | 283     | 208     | 188     | 179     | 94      |
| 8-10                 | 169    | 290     | 246     | 264     | 294          | 305     | 292     | 286     | 240     | 235     | 206     | 115     |
| 6, 7                 | 250    | 352     | 355     | 423     | 378          | 389     | 331     | 370     | 269     | 277     | 260     | 128     |
| 5                    | 182    | 276     | 257     | 290     | 244          | 288     | 273     | 254     | 219     | 217     | 188     | 70      |
|                      | 243    | 379     | 427     | 443     | 420          | 420     | 386     | 381     | 301     | 247     | 179     | 54      |
|                      |        |         |         |         | Whole Plate. |         |         |         |         |         |         |         |
| > 50                 | 103    | 147     | 134     | 125     | 144          | 189     | 169     | 159     | 149     | 170     | 216     | 203     |
| 30-49                | 381    | 543     | 501     | 523     | 583          | 611     | 606     | 679     | 702     | 796     | 845     | 527     |

TABLE II.—Number of Stars per square with Diameters greater than Successive Limits.

| Annulus                  | a    | b    | c    | d    | e            | f     | g     | h    | j    | k    | l    | m    |
|--------------------------|------|------|------|------|--------------|-------|-------|------|------|------|------|------|
| Diameter, 50 and greater | 3.4  | 3.6  | 3.1  | 2.3  | 3.6          | 4.0   | 4.1   | 2.7  | 2.9  | 3.1  | 4.7  | 5.7  |
| 30 "                     | 11.5 | 12.2 | 11.5 | 10.6 | 13.0         | 13.0  | 13.0  | 11.3 | 14.5 | 14.9 | 19.2 | 20.8 |
| 20 "                     | 28.5 | 27.3 | 28.8 | 30.0 | 33.8         | 37.7  | 34.1  | 32.9 | 37.5 | 33.7 | 39.4 | 36.1 |
| 15 "                     | 48.0 | 45.4 | 46.8 | 50.2 | 53.7         | 56.1  | 55.0  | 50.0 | 53.3 | 49.9 | 54.8 | 50.4 |
| 11 "                     | 66.8 | 65.1 | 66.9 | 67.8 | 73.5         | 76.2  | 74.7  | 72.3 | 72.9 | 66.3 | 71.4 | 68.1 |
| 8 "                      | 87.9 | 91.5 | 89.3 | 91.8 | 100.2        | 101.7 | 101.3 | 94.8 | 95.6 | 86.7 | 90.5 | 89.8 |
| 6 "                      | 119  | 123  | 121  | 130  | 134          | 134   | 131   | 124  | 121  | 111  | 114  | 114  |
| 5 "                      | 142  | 148  | 145  | 157  | 157          | 158   | 156   | 144  | 142  | 130  | 132  | 127  |
| 4 "                      | 172  | 183  | 184  | 197  | 195          | 193   | 191   | 174  | 170  | 151  | 148  | 137  |
|                          |      |      |      |      | Whole Plate. |       |       |      |      |      |      |      |
| 50                       | 3.2  | 3.3  | 3.0  | 2.8  | 3.3          | 3.9   | 3.9   | 3.2  | 3.7  | 3.7  | 4.9  | 8.4  |
| 30                       | 11.9 | 12.3 | 11.4 | 11.9 | 13.2         | 12.7  | 13.9  | 13.7 | 17.3 | 17.3 | 19.2 | 21.8 |

In order to find whether important systematic differences occurred between the four quarters of the plate, and in order to make the magnitude-corrections for the brighter stars depend on a more adequate amount of data, all four quarters were counted for diameters greater than 30 and greater than 50. The results for the whole plate are given in the last two lines of Tables I. and II.,\* and have been used in preference to the results for the single quarter in forming Tables III. and IV. The comparison of the four quarters of the plate is considered later.

For interpolation it is convenient to use the curve connecting  $\log N$  and  $\sqrt{d}$ , which is not far from linear. Here  $N$  denotes the relative number of stars with diameters greater than  $d$ . For a standard it is desirable to use a curve derived from a large number of stars to eliminate accidental irregularities; we have therefore used the mean of 49 squares  $x=7.14$ ,  $y=14.21$ . The points of this standard are as follows:—

| Diameter. | Number of Stars<br>(49 squares). | Log $N$<br>(per square). | $\sqrt{d} \dagger$ |
|-----------|----------------------------------|--------------------------|--------------------|
| 50        | 163                              | 0.522                    | 7.07               |
| 30        | 581                              | 1.074                    | 5.48               |
| 20        | 1459                             | 1.474                    | 4.47               |
| 15        | 2393                             | 1.689                    | 3.87               |
| 11        | 3339                             | 1.833                    | 3.32               |
| 8         | 4518                             | 1.965                    | 2.83               |
| 6         | 6176                             | 2.100                    | 2.45               |
| 5         | 7348                             | 2.176                    | 2.24               |
| 4         | 9156                             | 2.271                    | 2.00               |

Corresponding to any density of the stars in Table II., we read off from the curve the standard value of  $\sqrt{d}$ ; the difference between this and the actual  $\sqrt{d}$  is the required correction to  $\sqrt{d}$  depending on distance from the centre.

These corrections are given in Table III.

It is known that the relation between magnitude and diameter is approximately given by

$$m = a - 0.9 \sqrt{d}.$$

By multiplying the corrections to  $\sqrt{d}$  by  $-0.9$  we obtain the corrections to magnitude. The zero of these corrections is arbitrary (viz. they have been referred to the mean of a certain 49 squares). It is convenient to make the correction zero for a

\* The number of squares in the outer annuli is not quite the same for the top and bottom quarters of a plate, owing to the slope of the meridians which limit the area counted. For a bottom quarter the areas are  $g$ , 10.8;  $h$ , 12.1;  $j$ , 9.7;  $k$ , 11.5;  $l$ , 11.2;  $m$ , 6.8 squares; for the top quarters the areas are given in Table I.

† Strictly speaking  $(d + .5)$  has been used for  $d$ , as the diameters greater than 20 (inclusive) are those greater than 19.5. Evidently, for interpolation purposes, it does not matter whether we use  $\sqrt{d}$  or  $\sqrt{(d + .5)}$  for our curve.

TABLE III.—*Corrections to  $\sqrt{d}$ .*

| Distance from centre | 0'-17' | 17'-25' | 25'-31' | 31'-37' | 37'-41' | 41'-45' | 45'-49' | 49'-53' | 53'-57' | 57'-61' | 61'-68' | 68'-78' |
|----------------------|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Diameter 50          | - '03  | - '06   | + '04   | + '13   | - '06   | - '26   | - '26   | - '03   | - '19   | - '19   | - '52   | - '115  |
| 30                   | + '04  | + '01   | + '08   | + '04   | - '09   | - '04   | - '14   | - '13   | - '39   | - '39   | - '52   | - '66   |
| 20                   | + '04  | + '09   | + '03   | - '02   | - '15   | - '28   | - '16   | - '12   | - '27   | - '15   | - '33   | - '23   |
| 15                   | + '04  | + '08   | + '07   | - '02   | - '13   | - '20   | - '17   | - '02   | - '12   | - '01   | - '16   | - '03   |
| 11                   | + '04  | + '08   | + '04   | + '02   | - '11   | - '18   | - '14   | - '09   | - '10   | + '05   | - '07   | + '01   |
| 8                    | + '07  | '00     | + '04   | - '01   | - '11   | - '13   | - '12   | - '04   | - '05   | + '09   | + '02   | + '03   |
| 6                    | + '07  | + '02   | + '04   | - '04   | - '08   | - '08   | - '05   | + '02   | + '05   | + '15   | + '11   | + '12   |
| 5                    | + '06  | + '01   | + '04   | - '05   | - '05   | - '06   | - '05   | + '05   | + '07   | + '17   | + '15   | + '20   |
| 4                    | + '07  | '00     | '00     | - '08   | - '07   | - '06   | - '05   | + '06   | + '09   | + '23   | + '25   | + '35   |

TABLE IV.—*Corrections to Magnitude.*

| Diameter 50 | + '01 | + '04 | - '05 | - '13 | + '04 | + '22 | + '22 | + '01 | + '15 | + '15 | + '45 | + '1'02 |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| 30          | '00   | + '03 | - '04 | '00   | + '12 | + '07 | + '16 | + '15 | + 39  | + '39 | + '50 | + '63   |
| 20          | + '01 | - '04 | + '02 | + '06 | + '18 | + '30 | + '19 | + '15 | + '29 | + '18 | + '34 | + '25   |
| 15          | + '02 | - '02 | - '01 | + '07 | + '17 | + '23 | + '21 | + '07 | + '16 | + '06 | + '20 | + '08   |
| 11          | + '01 | - '03 | + '01 | + '03 | + '14 | + '21 | + '17 | + '13 | + '14 | '00   | + '11 | + '04   |
| 8           | - '03 | + '04 | '00   | + '04 | + '14 | + '15 | + '14 | + '07 | + '08 | - '04 | + '02 | + '01   |
| 6           | - '03 | + '02 | '00   | + '07 | + '11 | + '11 | + '08 | + '02 | - '01 | - '10 | - '06 | - '07   |
| 5           | - '02 | + '03 | '00   | + '08 | + '08 | + '09 | + '08 | - '01 | - '03 | - '12 | - '10 | - '14   |
| 4           | - '05 | + '02 | + '02 | + '09 | + '08 | + '07 | + '06 | - '04 | - '06 | - '19 | - '21 | - '30   |

circle of radius 30' about the origin, since the stars of the Harvard sequence, which are used as standards in the Greenwich work, are confined to this area. Making this adjustment, the corrections to the photographic magnitudes are given in Table IV.

The chief results to be learnt from Table IV. are:—

- (1) The general positive correction to the magnitudes of stars between 30' and 50' from the centre.
- (2) The (algebraically) increasing correction for bright images, and decreasing correction for faint images, beyond 50' from the centre.
- (3) The rapid numerical increase in the corrections, especially for the bright stars, beyond 60' from the centre, showing that is undesirable to use images beyond 60' for magnitude determinations.

The effect of the positive correction between 30' and 50' is that, if we use the Harvard sequence to determine magnitudes over a field of 57' radius, without correction, the resulting magnitudes will be in the mean 0<sup>m</sup>.08 too bright for diameters greater than 10. The error diminishes for the smaller diameters.

Smoothed corrections based on Table IV. are being applied in the determinations of magnitude for vol. iii. of the *Astrographic Catalogue*.

#### *Difference between the Four Quarters of the Plate.*

The numbers of stars counted in each quarter-plate are:—

| Diameter.      | Top left. | Bottom left. | Top right. | Bottom right. |
|----------------|-----------|--------------|------------|---------------|
| 50 and greater | 441       | 440          | 517        | 510           |
| 30     ,,      | 1697      | 1755         | 1957       | 1888          |

The areas counted in each quadrant are practically equal (actually the top quadrants contain 125.9 squares and the bottom 126.1).

It will be seen that there is a considerable difference, right *minus* left, in these numbers. Differences of this kind have been detected in previous investigations with other object-glasses. In the present case it appears to arise from a tilt of the object-glass, *i.e.* the axis of the object-glass passes to the left of the centre of the réseau. A tilt of the plate would doubtless have a similar effect, but the records of observations of tilt seem to forbid this explanation; the squaring-on of the object-glass is a much more difficult mechanical adjustment, and is the more likely cause of the difference.\* The effect of such a tilt is that the concentric

\* [Added May 17.] The present adjustment of the telescope has been examined. It was found that there was a tilt of 5'.5 in the object-glass, so that its axis passed through  $x=12.9$ ,  $y=14.0$  on the réseau. Reference to the Adjustment Book shows that the axis has remained quite constant in this position, at any rate since 1894; but no attempt to correct this had been made, as the approximation was amply sufficient for position determinations. The direction of this displacement agrees exactly with that indicated by the counts of stars. The tilt has now been corrected.

circles of equal definition are shifted towards one side of the réseau. If the shift amounts to a réseau interval, the definition at 55' on one side of the centre would only be equal to that at 65' on the other side, so that a considerable inequality in the star counts would be introduced. It is interesting to note that for magnitude work a much greater accuracy is desirable in this adjustment than is needed for position work.

From the above figures it is found that the mean difference between "left" and "right" corresponds to a difference of magnitude.

0<sup>m</sup>.16 for diameter 50

0<sup>m</sup>.11 for diameter 30.\*

This lateral asymmetry could best be taken into account by using as the argument for magnitude correction, not distance from the centre of the réseau, but distance from a point rather to one side of it; but such a correction would be inconvenient to apply to observations in bulk. It is clear from Table IV. that the difference arises mainly in the outermost annuli; and, if the measures are confined to images within 50' of the centre, as is usually done, the correction is perhaps negligible. It is, moreover, partly compensated when the mean is taken of images measured on overlapping plates; and in any case no general systematic error of the magnitudes can arise from it.

This investigation of the variation of the images with position on the plate has been made by Mr. Eddington, with a view to its use in the determination of the magnitudes of the *Astrographic Catalogue*, now being made under Mr. Hollis's direction.

### *On Correcting the Field of a Newtonian Telescope.*

By Professor R. A. Sampson, D.Sc., F.R.S.

In a paper recently published (*Phil. Trans*, vol. cexiii., p. 29), I have discussed the geometrical faults in the field of a Cassegrain reflector, and have showed how spherical aberration and coma might be suppressed, and the star-images reduced to very small circles upon a field strictly flat, by interposing in the beam a system of three lenses—one of which was a meniscus, silvered at the back and served as the reversing mirror—and by slightly figuring the great mirror to a curve intermediate between the circle and parabola. All the curves necessary are very shallow. Achromatism is secured by making all three lenses of the same glass and properly balancing their focal lengths, so that for any two rays, at the emergent principal focus, the position of focus and

\* We may expect that the effect would be reversed in sign for the very faint stars.

the size of image are exactly the same. The paper deals at some length with necessary details of thickness of the lenses, and so forth; but it is also shown that a close and reliable approximation may be obtained by abbreviated methods when these are suppressed. The present paper gives an account of the corresponding treatment of a Newtonian reflector by such methods, the lenses interposed being supposed thin. The details, therefore, are approximate; but according to my experience with the previous problem, they would represent closely the character of the solution.

Let the great mirror (o) be 40 inches aperture and 200 inches focal length; let the distance from it to the first lens (2) be 157 inches; the distance further to the second lens (4) be 6 inches, and thence to the third lens (6), 2 inches, after which the flat is placed. This choice of distances is more or less arbitrary. Let the focal length of the lens (4) be -40 inches, so that with the notation and unit (100 inches) employed in my paper above referred to

$$k_4 = \left(1 - \frac{1}{n}\right)(B_4 - B'_4) = +2.5000,$$

where  $B_4, B'_4$  are the curvatures of the two faces, and  $n^{-1}$  is the refractive index of the glass, taken at

$$n^{-1} = 1.5200.$$

In agreement with the theory given on p. 47 of the paper referred to, the values of  $k$  for the other two lenses at the adopted distances are

$$k_2 = \left(1 - \frac{1}{n}\right)(B_2 - B'_2) = -.5003, \quad k_6 = \left(1 - \frac{1}{n}\right)(B_6 - B'_6) = -.18405;$$

these three lenses form an achromatic combination, and we must now find how the curvatures must be distributed between the two faces of each so as best to correct the faults of the resultant field.

Employing the conditions as they are given in that paper for absence of spherical aberration, coma, and curvature of field, we must have at the principal focus,

$$\delta_1 G = 0, \quad \delta_2 G = 0, \quad \delta_3 G - \frac{1}{2} f'' \mathfrak{P} = 0,$$

where  $\mathfrak{P}$  is Petzval's expression, and we must express these in terms of the quantities  $q_2 \dots$ , or  $q_2 \dots$  where

$$q_2 = \left(1 + \frac{1}{n}\right)(B_2 + B'_2), \quad q_2 = q_2/k_2,$$

and in terms of the deformation of the great mirror from a spherical surface. I find by the methods illustrated on p. 55 of that paper, that the great mirror and the three lenses contribute at the emergent principal focus the following portions:—

*Great Mirror:—*

$$\delta_1 G = \dots + .0646 \epsilon_0, \quad \delta_2 G = +.2583, \quad \delta_3 G = 0,$$

where  $\epsilon_0 = 0$  for a parabolic figure, and  $= -1$  for a spherical one.

*First Lens:—*

$$\begin{array}{lll}
\delta_1 G = \dots - \cdot 0644 & - \cdot 0038 q_2 & - \cdot 0000, 5 q_2^2 \\
\delta_2 G = \dots - \cdot 1434 & - \cdot 0195 q_2 & - \cdot 0003, 7 q_2^2 \\
\delta_3 G = \dots - \cdot 3711 & - \cdot 0846 q_2 & - \cdot 0026, 9 q_2^2
\end{array}$$

*Second Lens:—*

$$\begin{array}{lll}
\delta_1 G = \dots + \cdot 1752 & - \cdot 0372 q_4 & + \cdot 0029, 9 q_4^2 \\
\delta_2 G = \dots + \cdot 6101 & - \cdot 1941 q_4 & + \cdot 0264, 9 q_4^2 \\
\delta_3 G = \dots + 5\cdot 6194 & - \cdot 5208 q_4 & + \cdot 2348, 1 q_4^2
\end{array}$$

*Third Lens:—*

$$\begin{array}{lll}
\delta_1 G = \dots - \cdot 0784 & - \cdot 0160 q_6 & - \cdot 0011, 1 q_6^2 \\
\delta_2 G = \dots - \cdot 1745 & - \cdot 0814 q_6 & - \cdot 0105, 5 q_6^2 \\
\delta_3 G = \dots - 2\cdot 5290 & - \cdot 1008 q_6 & - \cdot 1002, 6 q_6^2
\end{array}$$

*Further—*

$$f' = + 2\cdot 0667, \quad \mathfrak{P} = + \cdot 6048, \quad -\frac{1}{2}f'\mathfrak{P} = - \cdot 6250.$$

Equating to zero the sums of the corresponding parts, we have three equations, of which the constant parts are respectively

$$\begin{aligned}
\delta_1 G &= \dots + \cdot 0324, \quad \delta_2 G = \dots + \cdot 5505, \\
\delta_3 G - \frac{1}{2}f'\mathfrak{P} &= \dots + 2\cdot 0943,
\end{aligned}$$

and the variable parts in  $\epsilon_0, q$  are taken without change from the expressions above.

There are thus four quantities,  $\epsilon_0, q_2, q_4, q_6$ , to satisfy three equations, and, theoretically, any one quantity might be adopted and the other three determined accordingly, but in practice any utility the solution may possess depends upon the values determined lying between reasonable limits. Thus, for example,  $q = 10$  would make the greatest curvature of the lens about three times as great as that for an equi-curved lens of the same focal length for which  $q = 0$ , and this might be considered a practical limit. If then we suppose an existing parabolic reflector for which  $\epsilon_0 = 0$  and attempt to correct the field by adjusting the quantities  $q$ , it is soon clear that the moderate values of  $q$ , which will satisfy the second and third equations for coma and curvature, will leave the first equation under-corrected—that is in respect to spherical aberration. Hence it is not practicable to apply to an existing mirror, already parabolised to correct its spherical aberration, a system of three lenses arranged like these, and so further correct coma and curvature of the field. Nor does there appear to be a solution correcting completely spherical aberration and coma only, and curvature merely approximately. I shall then take the solution in which spherical aberration is corrected by an appropriate figuring of the great mirror. If we leave, say,  $q_2 = 0$ , we have the values

$$\epsilon_0 = + 0\cdot 944 \quad q_4 = + 0\cdot 3829 \quad q_6 + 3\cdot 9127$$

and these, in conjunction with the values given above,  $k_2, k_4, k_6$ , give the curvatures

$$\begin{array}{lll} B_2 = +.4811 & B_4 = -2.2636 & B_6 = +0.3409 \\ B_2' = -.4811 & B_4' = +2.5442 & B_6' = -3.1985, \end{array}$$

and for the radii, with unit 1 inch,

$$\begin{array}{lll} R_2 = +207.86 & R_4 = -44.18 & R_6 = +293.34 \\ R_2' = -207.86 & R_4' = +39.31 & R_6' = -31.27. \end{array}$$

The figure of the great mirror is nearly as far beyond the paraboloid as the paraboloid is beyond the sphere.

With these data the radii of the circular images of points, at angular distance  $\tan^{-1}\beta$  from the centre of the field, would be  $0''.312 \times (100\beta)^2$ , or at  $\beta = .01 = \tan 34'.4$ ,  $0''.31$ . The remaining fault of the field, distortion, which has been left upon one side, may be calculated readily; by the methods illustrated above, I find  $\delta_3 H = +34'.33$ , and the distortional displacement in a field  $\beta$  equal to

$$+1''.71 \times (100\beta)^3.$$

This displacement would not vitiate the star-image in any way, which would be shifted as a whole, and it would be allowed for in any measurements, along with other unavoidable corrections, by calculation.

All the quantities specified above are within easy reach of practice, and practically the most vital question would be that of the loss of light in passage through three lenses.

In the uncorrected Newtonian of a field  $\beta = .01$ , the light collected by the great mirror would be distributed over, but most along the edges of, a fan-shaped area about  $.16''$  long and of greatest breadth about  $11''$ ; and though distortion is absent, the tip of this fan would not be strictly at the normal image point, because that belongs only to the central zones, and these are cut out by the flat. The result is a very serious falling off in the efficiency, as a light collector, of a Newtonian of large aperture-ratio, outside the centre of the field, as well as an inherent fault in respect to measurement.

*A Preliminary Discussion of the Discordance between the Observed and Predicted Positions of Jupiter's Eighth Satellite.* By John Jackson, M.A., B.Sc., B.A.

This paper is to be regarded as merely tentative. The ephemeris\* employed is that of Cowell, Crommelin, and Davidson, based on the observations made during the opposition of 1908 and the early part of the opposition of 1909. As the predictions have recently proved to be considerably in error, it has been considered advisable to get preliminary corrections before proceeding to a general discussion of the orbit.

\* *M.N.*, lxix. 421; lxxi. 50.

A glance at the discordance shows that the predicted position is ahead of the observed, and that a large part of the residuals can be accounted for by decreasing the mean motion of the satellite. As the mean motion is closely related to the mean distance and the mass of the primary, the mean distance is also considered to be in error. In this paper, therefore, it is assumed that the initial orbit of the ephemeris is correct in shape and position, and that only the mean motion and the mean distance require correction.

The following plan has been adopted. The ephemeris gives the Jovicentric equatorial co-ordinates  $(x, y, z)$  at intervals of sixteen days. From these, by mechanical differentiation, the velocities  $(\dot{x}, \dot{y}, \dot{z})$  in units of sixteen days have been derived. Let  $r, a, \delta$  be the geocentric radius vector, right ascension, and declination derived from  $(x, y, z)$ , and let  $r + dr, a + da, \delta + d\delta$  be related in the same way to  $\left(x + \frac{\dot{x}}{16}, y + \frac{\dot{y}}{16}, z + \frac{\dot{z}}{16}\right)$ . To the first order,  $da, d\delta$  are given by

$$r \cos \delta da = -x \frac{206265}{15 \times 16} \sin a + y \frac{206265}{15 \times 16} \cos a,$$

$$r l \delta'' = -x \frac{206265}{16} \cos a \sin \delta - y \frac{206265}{16} \sin a \sin \delta + z \frac{206265}{16} \cos \delta.$$

It will be seen that  $da, d\delta$  give the corrections to the ephemeris on the assumption that the Jovicentric positions there used are one day behind the true positions. Or, again, if at time  $t_0$  the ephemeris and observations agree, while the mean motion  $n$  of the ephemeris requires a correction  $\delta n$ , the corrections to the  $a, \delta$  of ephemeris at time  $t$ , measured in years from  $t_0$ , will be  $da(t - t_0) \times 365 \frac{1}{4} \frac{\delta n}{n}, d\delta(t - t_0) \times 365 \frac{1}{4} \frac{\delta n}{n}$ . A correction to the mean distance of the satellite from Jupiter will be very nearly allowed for by assuming corrections proportional to  $a_{\text{viii}} - a_J$ , and  $\delta_{\text{viii}} - \delta_J$ .

On the assumptions made we may therefore write

$$\Delta a = x a' + y da(t - t_0),$$

$$\Delta \delta = x \delta' + y d\delta(t - t_0),$$

where  $\Delta a, \Delta \delta$  are the residuals in the sense calculated - observed;  $a', \delta'$  are  $a_{\text{viii}} - a_J, \delta_{\text{viii}} - \delta_J$ . The corrections to  $a, n$ , the mean distance and mean motion of the satellite, are then  $-ax, -\frac{1}{365 \frac{1}{4}} ny$ . [The negative signs are due to the fact that  $\Delta a, \Delta \delta$  are taken in the sense stated above.] We assume  $t, t_0$  to be measured from 1908.000, and write  $-yt_0 = z$ , so that

$$\Delta a = x a' + y t . da + z da,$$

with a similar equation for  $\Delta \delta$ .

In the subsequent analysis all the available observations\* have

\* *M.N.*, lxix. 429, 675; lxxi. 63; lxxii. 7; *Egyptian Survey Papers*, No. 7; *Lick Obs. Bull.*, 217.

been used; the Greenwich observations of 1908, 1909, 1910; the Helwân observations of 1911, 1912; two observations at Heidelberg in 1908; three in 1908, and seven in 1912 at Lick. A new reduction of all these observations has been made, allowing for difference in the aberration times of Jupiter and its satellite and for the differential precession and nutation from equinox of date to equinox of ephemeris. These corrections have been neglected in previous reductions, but in 1912 they amounted to more than  $1''\cdot 0$ .

The following table gives the normal places that have been used; the numbers in brackets following the dates give the number of observations on which the residuals have been based. The table also gives the corresponding values of  $a'$ ,  $tda$ ,  $da$ ;  $\delta'$ ,  $td\delta$ ,  $d\delta$ .

|              | $\Delta a$ . | $a'$ .       | $tda$ .      | $da$ .       | $\Delta \delta$ . | $\delta'$ . | $td\delta$ . | $d\delta$ . |
|--------------|--------------|--------------|--------------|--------------|-------------------|-------------|--------------|-------------|
| 1908.        | <sup>s</sup> | <sup>s</sup> | <sup>s</sup> | <sup>s</sup> |                   |             |              |             |
| Jan. 28 (2)  | +0'245       | - 7          | + '255       | +3'493       | - 0'9             | -43'2       | + 0'9        | +12'51      |
| Feb. 3 (2)   | + '245       | + 7          | + '332       | +3'499       | - 0'2             | -40'5       | + 1'3        | +13'56      |
| 25 (5)       | + '126       | + 54         | + '513       | +3'422       | + 0'4             | -33'0       | + 2'3        | +15'44      |
| Mar. 6 (2)   | + '230       | + 78         | + '590       | +3'331       | + 2'3             | -29'0       | + 2'8        | +16'10      |
| 30 (5)       | + '144       | +138         | + '725       | +2'985       | + 1'1             | -20'1       | + 4'1        | +16'75      |
| Apr. 27 (2)  | + '125       | +207         | + '767       | +2'397       | + 0'3             | -11'8       | + 5'3        | +16'46      |
| 1909.        |              |              |              |              |                   |             |              |             |
| Jan. 17 (2½) | + '123       | -383         | - 1'857      | -1'779       | + 2'1             | +13'7       | - 27'1       | -25'95      |
| Feb. 16 (6)  | + '305       | -447         | - '833       | - '739       | + 0'4             | - 0'7       | - 32'2       | -28'53      |
| Apr. 14 (3½) | + '323       | -434         | + 1'071      | + '835       | - 0'7             | -25'2       | - 30'4       | -23'68      |
| May 8 (1)    | + '160       | -382         | + 1'801      | +1'335       | - 0'8             | -31'7       | - 27'1       | -20'08      |
| 1910.        |              |              |              |              |                   |             |              |             |
| Jan. 20 (½)  | + '090       | +449         | + 2'371      | +1'155       | + 5'3             | -77'1       | + 31'1       | +15'14      |
| Feb. 14 (3)  | + '036       | +508         | + 1'799      | +0'848       | + 6'3             | -75'3       | + 41'8       | +19'72      |
| Apr. 28 (4)  | - '310       | +536         | - 0'592      | -0'255       | + 9'3             | -47'8       | + 65'5       | +28'22      |
| 1911.        |              |              |              |              |                   |             |              |             |
| Feb. 5 (1)   | + '770       | -152         | + 7'589      | +2'451       | -13'6             | -27'2       | -142'1       | -45'91      |
| 27 (½)       | + '970       | - 94         | + 9'660      | +3'061       | -11'9             | -46'4       | -134'8       | -42'70      |
| Mar. 9 (½)   | + '970       | - 65         | +10'335      | +3'247       | -12'3             | -54'9       | -130'6       | -40'92      |
| 30 (3)       | +1'087       | - 6          | +11'275      | +3'479       | -11'8             | -71'0       | -118'3       | -36'51      |
| Apr. 26 (4)  | +1'106       | + 65         | +11'639      | +3'511       | - 9'3             | -87'2       | - 98'9       | -29'83      |
| May 25 (3)   | +1'053       | +130         | +11'125      | +3'278       | - 6'4             | -96'6       | - 74'5       | -21'94      |
| June 25 (4)  | +0'916       | +196         | + 9'908      | +2'848       | - 3'2             | -98'3       | - 48'0       | -13'81      |
| 1912.        |              |              |              |              |                   |             |              |             |
| Mar. 23 (3)  | -1'716       | +405         | - 9'542      | -2'259       | +14'8             | -16'0       | +109'7       | +25'98      |
| Apr. 22 (2)  | -2'121       | +363         | -12'724      | -2'955       | +16'6             | - 3'4       | +125'4       | +29'12      |
| May 20 (3)   | -2'517       | +309         | -15'560      | -3'550       | +19'9             | + 8'8       | +136'7       | +31'19      |
| June 8 (5)   | -2'590*      | +259         | -17'306      | -3'903       | +19'6             | +17'2       | +138'9       | +31'32      |
| 10 (2)       | -2'550       | +253         | -17'471      | -3'935       | +18'9             | +18'1       | +138'7       | +31'24      |
| 16 (2)       | -2'545       | +234         | -17'943      | -4'025       | +17'8             | +20'7       | +137'7       | +30'89      |

\* This is obtained from five Lick observations: -2'45, -2'82, -2'59, -2'60, -2'51.

The table gives the data from which normal equations in  $x, y, z$  may be formed. Much of the arithmetic was avoided and practically all the accuracy of the method of least-square solution was obtained by using quantities nearly equal to product of number of observations and coefficient of unknown. Thus, for the last year's observations in right ascension in forming the normal equation for  $x$ , instead of multipliers 1215, 726, 927, 1036, 506, 468, the nearest exact hundred was used—i.e. 1200, 700, 900, 1000, 500, 500. This allowed the arithmetic to be done mentally. The resulting equations had not quite the usual symmetrical form.

We now write  $x' = 100x$  in forming the approximately normal equations in  $a$ , and  $x'' = 60x$  in forming those in  $\delta$ . The following equations result:—

$$\left. \begin{aligned} + 632.7x' - 502.2y - 40.1z &= -114.53 \\ - 528.1x' + 5660.9y + 1473.0z &= +758.03 \\ - 51.3x' + 1481.4y + 569.3z &= +196.90 \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} + 17863x'' + 11327y + 2284z &= +939.5 \\ + 11330x'' + 46960y + 13179z &= +5650.9 \\ + 2271x'' + 13282y + 4662z &= +1486.6 \end{aligned} \right\} \quad (2)$$

The solutions of (1) and (2) are respectively—

$$\begin{aligned} x &= -.000857, & y &= +.11176, & z &= +.0321; \\ x &= -.000621, & y &= +.1733, & z &= -.1567. \end{aligned}$$

This result seems to indicate that the original hypothesis is considerably in error. The first value of  $z$  corresponds to  $t_0 = 1907.726$ , the second to 1908.904. Now, on the hypothesis it should be expected that  $t_0$  would fall between the two sets of observations on which the initial orbit was based—say, 1908.6 or 1908.7. Moreover, the equations are not suitable for determining  $y$  and  $z$ , as the second equation approximates to 3.5 times the third. We therefore assume  $z$  to be known and determine  $x, y$  from the first two equations and calculate the right-hand side of the third to see how closely the assumption agrees with the observations.

In the analysis of  $\Delta a$  we get

$$\begin{aligned} \text{with } t_0 &= 1908.7, & x &= -.000638, & y &= +.1564, \\ t_0 &= 1908.6, & x &= -.000665, & y &= +.1515, \end{aligned}$$

and the right-hand side of the third equation becomes in these two cases 172.95, 176.17, instead of 196.90.

Similarly  $\Delta \delta$  gives

$$\begin{aligned} \text{with } t_0 &= 1908.7, & x &= -.000580, & y &= +.1601, \\ t_0 &= 1908.6, & x &= -.000558, & y &= +.1544, \end{aligned}$$

with the right-hand side of the third equation, 1525.8, 1542.0, instead of 1486.6.

It will be seen that the agreement in the results for  $\Delta\alpha$ ,  $\Delta\delta$  is now much closer.

With the results obtained we may calculate rough approximations to the orbit at or near the times of the observations. We will write—

$$\Delta\alpha_1 = -\cdot00857\alpha' + da(\cdot1176t + \cdot0321),$$

$$\Delta\alpha_2 = -\cdot00638\alpha' + da\cdot1564(t - \cdot1908\cdot7),$$

$$\Delta\delta_1 = -\cdot00621\delta' + d\delta(\cdot1733t - \cdot1567).$$

The following table gives approximate values:—

|         | $\Delta\alpha_1$ . | $\Delta\alpha_2$ . | $\Delta\delta_1$ . |
|---------|--------------------|--------------------|--------------------|
| 1908.   |                    |                    |                    |
| Feb. 5  | + <sup>s</sup> '14 | - <sup>s</sup> '34 | - 0''4             |
| Mar. 8  | + '11              | - '33              | - 1'0              |
| Apr. 25 | - '01              | - '29              | - 1'2              |
| 1909.   |                    |                    |                    |
| Jan. 22 | + '09              | + '16              | - 1'1              |
| Apr. 12 | + '52              | + '35              | - 0'6              |
| May 14  | + '59              | + '38              | - 0'4              |
| 1910.   |                    |                    |                    |
| Jan. 25 | - '09              | - '06              | + 6'1              |
| Feb. 10 | - '17              | - '13              | + 6'8              |
| May 1   | - '55              | - '42              | + 8'7              |
| 1911.   |                    |                    |                    |
| Mar. 1  | + 1'33             | + 1'25             | - 14'8             |
| Apr. 2  | + 1'45             | + 1'39             | - 11'8             |
| May 4   | + 1'40             | + 1'38             | - 8'3              |
| 20      | + 1'33             | + 1'34             | - 6'4              |
| June 5  | + 1'23             | + 1'24             | - 4'6              |
| 1912.   |                    |                    |                    |
| Mar. 19 | - 1'50             | - 1'45             | + 15'3             |
| Apr. 20 | - 1'88             | - 1'87             | + 17'2             |
| May 22  | - 2'13             | - 2'15             | + 19'6             |
| June 7  | - 2'39             | - 2'44             | + 18'5             |
| 23      | - 2'48             | - 2'57             | + 17'9             |

If we compare the calculated residuals with  $\Delta\alpha$ ,  $\Delta\delta$ , we get roughly—

|                                 | 1908'2. | 1909'2. | 1910'2. | 1911'3. | 1912'4. |
|---------------------------------|---------|---------|---------|---------|---------|
| $\Delta\alpha_1 - \Delta\alpha$ | - 8'10  | + '20   | - '20   | + '35   | + '20   |
| $\Delta\alpha_2 - \Delta\alpha$ | - 8'50  | + '05   | - '10   | + '30   | + '20   |
| $\Delta\delta_1 - \Delta\delta$ | - 2''0  | - 1''0  | 0       | 0       | 0       |

The agreement in declination is seen to be very close. In right ascension the difference appears to be periodic. It may be

remarked that the period between the oppositions of Jupiter is practically one-half the periodic time of the satellite, and thus errors in the shape and position of the initial orbit would cause such an effect as that noted in right ascension. The satellite was at apojove about 1911·84, which may help to explain the sign in the last column. The apojove of 1909·93 would have a less marked effect.

We may conclude from the above discussion that the mean motion  $n$  requires a correction of about  $-\frac{155}{365\frac{1}{4}}n$ , or  $-.00043n$ .

Of this there can be no serious doubt. The correction to the mean distance is much less certain, and the above seems to indicate a correction to  $a$  of about  $+.00061a$ . Now, an analysis of the osculating ellipse of the ephemeris at intervals of eighty days has led to a mean distance of 156707 astronomical units and a mean motion of 1748''·94 per mean solar day. [This work has not yet been published.] For undisturbed elliptic motion we have, in the usual notation,  $k^2(m_J + m_{\text{viii}}) = n^2 a^3$ . In this case we get—

$$n^2 a^3 = 0.979251 k^2 m_J.$$

We may therefore assume that corrections  $\delta n$ ,  $\delta a$ ,  $\delta m_J$  to  $n$ ,  $a$ ,  $m_J$  are related by

$$0.98 \frac{\delta m_J}{m_J} = 2 \frac{\delta n}{n} + 3 \frac{\delta a}{a}.$$

$$\text{If } \frac{\delta n}{n} = -.00043, \quad \frac{\delta a}{a} = +.00061,$$

$$\begin{aligned} \frac{\delta m_J}{m_J} &= (+.00183 - .00086) \times 1.02 \\ &= +.00099. \end{aligned}$$

A correction of 1 part in 1000 to the mass of Jupiter must be too large. We therefore conclude that  $\delta a$  has not been determined with sufficient accuracy. An error in the eccentricity of the initial orbit might vitiate the determination of  $a$ .

We are now in a position to calculate corrections to the ephemeris. We may, however, do so in various ways. It happens that at the opposition of this year the satellite has a very large elongation from Jupiter. On account of the uncertainty in the correction to the mean distance, it is therefore peculiarly difficult to determine the corrections to the ephemeris. Conversely, the occasion is peculiarly suitable for the determination of the mean distance.

We calculate the following corrections to the ephemeris:—

$$-\Delta a_2 = +.000638a' - .1564da(t - 1908.7)$$

$$-\Delta a_3 = +.000319a' - .1564da(t - 1908.7)$$

$$-\Delta \delta_1 = +.000621\delta' - .1733da(t - 1908.0) + .1567da.$$

| 1913.  | $-\Delta a_2$ | $-\Delta a_3$ | $-\Delta \delta_1$ |
|--------|---------------|---------------|--------------------|
|        | <sup>s</sup>  | <sup>s</sup>  |                    |
| May 1  | - '741        | - '913        | + 4''97            |
| 17     | - '491        | - '679        | + 3'34             |
| June 2 | - '261        | - '462        | + 1'63             |
| 18     | - '054        | - '264        | - 0'15             |
| July 4 | + '135        | - '081        | - 1'96             |
| 20     | + '308        | + '092        | - 3'73             |
| Aug. 5 | + '477        | + '265        | - 5'40             |
| 21     | + '645        | + '441        | - 6'89             |

The corrections to  $a$ ,  $\delta$  are much smaller than those for 1911, 1912. This is due to the fact that the motion of the satellite during the opposition of this year is approximately along the radius vector away from the earth, so that a considerable change in the Jovicentric longitude has little effect on the geocentric position.

So far we have considered that the change to the ephemeris resulting from changes  $\delta a$ ,  $\delta n$  could be allowed for in the Jovicentric positions by altering the unit of the time and adopting, if necessary, a new mass of Jupiter. This would probably be sufficiently accurate in ordinary satellite theories; but in this case the solar perturbations are so great that a change in the mean motion of the order here indicated might necessitate, at least during the later years of the ephemeris, a complete recalculation of these perturbations. If the necessary reduction to the mean motion is  $\cdot 00043n$ , then in five years the mean Jovicentric longitude of the ephemeris will be about  $23'$  in excess of the actual value. Otherwise stated, if the unit of time be suitably altered to allow for this, the Sun's longitude used in the calculation of the perturbations will require an increase of about  $4'$ . We may consider figs. 1 and 2 to show the relative configurations of the Sun, Jupiter, and the satellite about 1913 January 1 and 1913 July 20.

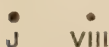


FIG. 1.

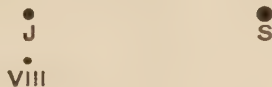


FIG. 2.

Let  $r$ ,  $r'$  be the distances of the satellite and the Sun from Jupiter, and let  $m$ ,  $m'$  be the masses of Jupiter and the Sun. The solar perturbations in the two cases may with sufficient accuracy be treated as:—

$$(1) \text{ an acceleration } -\frac{2m'r}{r'^3} - \frac{3m'r^2}{r'^4} \text{ towards Jupiter,}$$

$$(2) \text{ an acceleration } +\frac{m'r}{r'^3} \text{ towards Jupiter, and an accelera-}$$

tion  $+ \frac{3}{2} m' \frac{r^2}{r'^4}$  perpendicular to this in the direction of motion.

Now suppose the Sun to have moved through an angle  $\theta = 4'$  relative to Jupiter. We get, instead of the above—

(1) an acceleration  $-\frac{2m'r}{r'^3} - \frac{3m'r^2}{r'^4}$  towards Jupiter, and an acceleration  $-3m' \frac{r}{r'^3} \theta$  perpendicular to the radius vector;

(2) an acceleration  $+\frac{m'r}{r'^3}$  towards Jupiter, and an acceleration  $+\frac{3}{2} m' \frac{r}{r'^3} \left( \frac{r}{r'} + 2\theta \right)$  perpendicular to the radius vector.

We see that in the two cases accelerations perpendicular to the Jovicentric radius vector, of magnitude  $3m' \frac{r}{r'^3} \theta$  arise, but with opposite signs. The ratio of this to the acceleration caused by Jupiter's attraction is  $3 \frac{m'}{m} \left( \frac{r}{r'} \right)^3 \theta$ ; the ratio to that caused by the Sun's attraction is  $\frac{3}{2} \theta$  in the first case,  $3\theta$  in the second. On 1913 January 1  $\frac{r}{r'} = .0231$ , and on 1913 July 20  $\frac{r}{r'} = .0413$ , approximately, giving  $3 \frac{m'}{m} \left( \frac{r}{r'} \right)^3 \theta$  the values .000045, .000258 if  $\theta = 4'$ . The corrections to the perturbations are .0017, .0035 of their whole amount. This must be regarded as a considerable correction; its amount will increase steadily with the time.

Besides these changes all the perturbations require to be increased, as the mean distance of the satellite requires to be increased. If the adopted mass of Jupiter is correct, the mean distance will require to be increased by about 1 part in 3000; the perturbative force will therefore require to be increased by 1 part in 1000, and, as the time unit is altered to allow for decrease in mean motion, the perturbations will all require to be increased by about 1 part in 700.

The ephemeris has been constructed to the end of 1915, by which time  $\theta$  may be about  $7'$ . Hence towards the end of the period covered by the ephemeris we may expect to find it necessary to introduce changes in the perturbations equal to .0006 of the whole attraction of Jupiter. The mass of Jupiter is said to be known to .0002 of its whole amount. Unless the ephemeris can be corrected for the effects here indicated, it will clearly be useless for the determination of Jupiter's mass. In fact, these errors in the perturbations may be in large measure responsible for the

improbable correction to Jupiter's mass obtained above. It may also be quoted here that the authors of the ephemeris state that the cumulative errors of their method might even by 1910 amount to  $1''\cdot 0$ . The theory of the motion will require to be much improved before the observations will enable us to get the necessary correction to Jupiter's mass, although there can be no doubt that if theoretical difficulties can be surmounted this satellite will give a far better determination of Jupiter's mass than is obtainable by any other method.

*Note on Observations.*—It has been remarked that the period of the satellite is very near to twice the interval between two oppositions of Jupiter. It thus happens that the positions in which the satellite is observed will for several years be more or less confined to two regions of the orbit, in each of which it is observed at intervals of about twenty-six months. Hence it is desirable that observations should be made as long before and as long after opposition as is possible. It will much facilitate the determination of the orbit if three normal places can be obtained—say, one two months before opposition, one near opposition, and one two months after opposition. The opposition of this year will be peculiarly favourable for the determination of the distance of the satellite from Jupiter, as the satellite is at apojove about September 14, while the direction to the Earth is nearly perpendicular to the radius vector from Jupiter. The result is an elongation of more than  $3^\circ$  during July. During 1914 the satellite will be within  $2^\circ$  of Jupiter, and during 1915 it will be little more than  $1^\circ$  away.

1913 May 7.

*Note on the Pressure of Radiation on a Small Reflecting Sphere.*  
By J. Proudman.

(Communicated by T. J. P.A. Bromwich, Sc.D., F.R.S.)

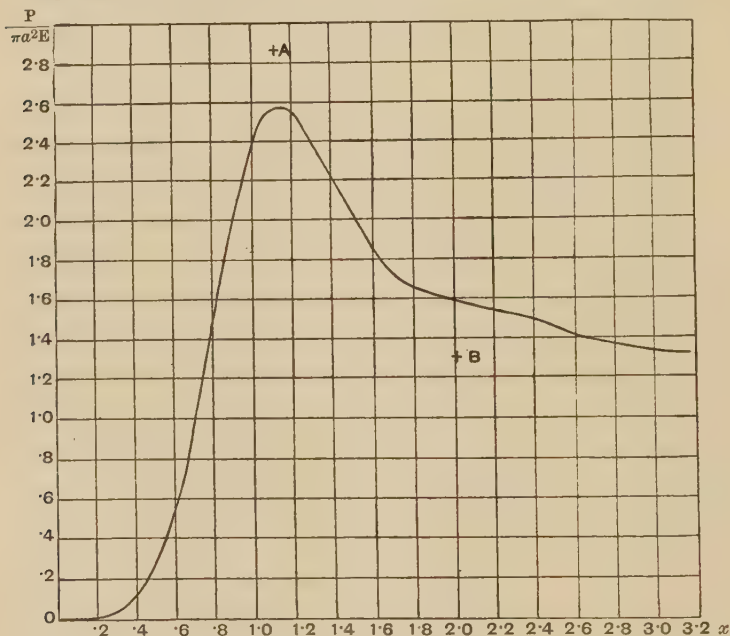
1. This problem has been discussed by Schwarzschild\* and Professor Nicholson,† but their numerical results are not in complete agreement. The analysis of the latter is much the shorter, and it appeared likely that his results were nearer the truth. Dr. Bromwich, however, was in possession of an independent treatment which showed that this was not the case as regards the magnitude of the maximum pressure. He informed me of this, and suggested that I should make a fresh calculation, very kindly showing me his results. I have worked through all the cases considered by Professor Nicholson, together with some additional and intermediate ones, and the results are substantially in agreement except as regards the maximum pressure. The present note contains the results referring to this maximum, making the determination much more precise than was done by Schwarzschild.

\* *Sitz. d. Math. Phys. Classe zu Munchen*, 1901-2, p. 293.

† *Monthly Notices of the R.A.S.*, vol. lxx. p. 544.

A graph is appended showing the results over the whole range considered.

The method used is practically the same as that of Professor Nicholson, but the analysis (obtained from the lectures of Dr. Bromwich) is rather shorter. The final formulæ are in agreement, though their forms are different.



2. Take spherical polar co-ordinates  $r, \theta, \phi$ , and let a plane wave train, containing the time factor  $e^{ikt}$ , and polarised in the plane  $\phi = \frac{1}{2}\pi$ , travel along the negative direction of the axis  $\theta = 0$ . Here  $c$  is the velocity of radiation, and  $2\pi/\kappa$  is the wave-length.

Let  $X, Y, Z$  and  $\alpha, \beta, \gamma$  denote respectively the components of the electric and magnetic forces, in the directions of  $r, \theta, \phi$  increasing. Then for the *incident wave*, with a suitable choice of the amplitude, we shall have

$$X = -\sin\theta \cos\phi e^{ikr \cos\theta}, \quad c\alpha = \sin\theta \sin\phi e^{ikr \cos\theta}, \quad (1)$$

omitting the time factor. From these we obtain for the *total disturbance* outside the sphere, on the assumption of perfect reflection,

$$X = -\frac{\cos\phi}{(\kappa r)^2} \sum_{n=1}^{\infty} i^{n-1} (2n+1) \left\{ S_n(\kappa r) - \frac{S_n'(\kappa a)}{E_n'(\kappa a)} E_n(\kappa r) \right\} \sin\theta P_n'(\cos\theta),$$

$$c\beta = \frac{\sin\phi}{\kappa r} \sum_{n=1}^{\infty} i^{n-1} \frac{2n+1}{n(n+1)} \left\{ S_n'(\kappa r) - \frac{S_n(\kappa a)}{E_n(\kappa a)} E_n'(\kappa r) \right\} \frac{d}{d\theta} \left\{ \sin\theta P_n'(\cos\theta) \right\},$$

$$+ \frac{\sin\phi}{\kappa r} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left\{ S_n(\kappa r) - \frac{S_n'(\kappa a)}{E_n'(\kappa a)} E_n(\kappa r) \right\} P_n'(\cos\theta),$$

$$c\gamma = \frac{\cos \phi}{\kappa r} \sum_{n=1}^{\infty} i^{n-1} \frac{2n+1}{n(n+1)} \left\{ S_n'(\kappa r) - \frac{S_n(\kappa a)}{E_n(\kappa a)} E_n'(\kappa r) \right\} P_n'(\cos \theta) \\ + \frac{\cos \phi}{\kappa r} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} \left\{ S_n(\kappa r) - \frac{S_n'(\kappa a)}{E_n'(\kappa a)} E_n(\kappa r) \right\} \frac{d}{d\theta} \left\{ \sin \theta P_n'(\cos \theta) \right\} \quad (2)$$

where  $r=a$  is the equation of the sphere, these being the only components we require.

Here

$$S_n(z) = z^{n+1} \left( -\frac{1}{z} \frac{d}{dz} \right)^n \left( \frac{\sin z}{z} \right), \quad E_n(z) = z^{n+1} \left( -\frac{1}{z} \frac{d}{dz} \right)^n \left( \frac{e^{-iz}}{z} \right),$$

from which can be deduced

$$S_n'(z)E_n(z) - S_n(z)E_n'(z) = 1.$$

Taking now only the real parts of (2), the average pressure of the radiation at any point of the sphere is given by

$$\frac{1}{2} E \{ (c\beta)^2 + (c\gamma)^2 - X^2 \},$$

on putting  $r=a$ , where  $E$  denotes the total average energy per unit volume of the incident wave.

On integrating over the sphere (using the formulæ of Nicholson's paper) the component in the direction of the incident train, the resultant pressure  $P$  in this direction is given by

$$\frac{P}{\pi a^2 E} = \xi + \eta + \zeta, \quad (3)$$

where

$$\left. \begin{aligned} \xi &= \frac{1}{x^2} \sum_{n=1}^{\infty} \frac{2(2n+1)}{n(n+1)} \frac{1}{|E_n(x)|^2 |E_n'(x)|^2}, \\ \eta &= \frac{1}{x^2} \sum_{n=1}^{\infty} \frac{2n(n+2)}{n+1} \frac{1}{|E_n(x)|^2 |E_{n+1}(x)|^2}, \\ \zeta &= \frac{1}{x^2} \sum_{n=1}^{\infty} \frac{2n(n+2)}{n+1} \left\{ \frac{(n+1)^2}{x^2} - 1 \right\}^2 \frac{1}{|E_n'(x)|^2 |E_{n+1}'(x)|^2}, \end{aligned} \right\} \quad (4)^*$$

$x$  being written for  $\kappa a$ .

3. Now it is easily verified that

$$|E_1(x)|^2 = \frac{1}{x^2} + 1, \quad |E_1'(x)|^2 = \frac{1}{x^4} - \frac{1}{x^2} + 1,$$

$$|E_2(x)|^2 = \frac{9}{x^4} + \frac{3}{x^2} + 1, \quad |E_2'(x)|^2 = \frac{36}{x^6} - \frac{3}{x^2} + 1,$$

$$|E_3(x)|^2 = \frac{225}{x^6} + \frac{45}{x^4} + \frac{6}{x^2} + 1, \quad |E_3'(x)|^2 = \frac{2025}{x^8} + \frac{135}{x^6} - \frac{9}{x^4} - \frac{6}{x^2} + 1.$$

General formulæ can be given, but these are found to be sufficient for the accuracy of the last column in the following table.

In this table 1.054 and 1.118 are the approximate values of  $\sqrt{(10/9)}$  and  $\sqrt{(10/8)}$  respectively.

\* These series are equivalent respectively to the series  $S_2$ ,  $\eta$ ,  $\zeta$  of Nicholson; for  $|E_n(x)|^2$  he writes  $R_n$ , and for  $|E_n'(x)|^2$  he writes  $\sec^2 X_n/R_n$ .

| $x$   | $ E_1(x) ^2$ | $ E_2(x) ^2$ | $ E_3(x) ^2$ | $ E_1(x) ^2$ | $ E_2(x) ^2$ | $ E_3(x) ^2$ | $\xi$  | $\eta$ | $\zeta$ | $\frac{P}{\pi a^2 E}$ |
|-------|--------------|--------------|--------------|--------------|--------------|--------------|--------|--------|---------|-----------------------|
| 1.000 | 2.0000       | 13.000       | 277.0        | 1.00000      | 34.00        | 2146.0       | 1.5038 | .1169  | .7988   | 2.419 *               |
| 1.054 | 1.9000       | 10.990       | 206.9        | .91000       | 24.54        | 1415.3       | 1.5672 | .1314  | .8242   | 2.523 *               |
| 1.100 | 1.8265       | 9.627        | 163.7        | .85657       | 18.84        | 1010.8       | 1.5920 | .1438  | .8263   | 2.562 *               |
| 1.118 | 1.8000       | 9.160        | 149.8        | .84000       | 17.03        | 889.0        | 1.5958 | .1487  | .8228   | 2.567 *               |
| 1.120 | 1.7972       | 9.111        | 148.4        | .83832       | 16.85        | 876.7        | 1.5960 | .1492  | .8224   | 2.568                 |
| 1.125 | 1.7901       | 8.989        | 144.8        | .83418       | 16.39        | 846.5        | 1.5963 | .1505  | .8206   | 2.567                 |
| 1.150 | 1.7561       | 8.414        | 128.5        | .81561       | 14.29        | 711.6        | 1.5943 | .1572  | .8109   | 2.562                 |
| 1.200 | 1.6944       | 7.424        | 102.2        | .78781       | 10.97        | 508.7        | 1.5746 | .1705  | .7800   | 2.525 †               |

\* The first four results agree with those previously determined by Dr. Bromwich.

† The first and last results agree with those given by Professor Nicholson.

In the range of the above table the only case considered by Schwarzschild is  $x = 1$ . For the function tabulated in the last column he gives 2.42. The cases considered by Nicholson are  $x = 1, 1.1,$  and  $1.2$ ; his results are respectively 2.41945, 2.856, and 2.525.

It is clear from the table that the maximum occurs near  $x = 1.12$ , and that its value cannot be far from 2.568.

The graph shows the results from  $x = 0$  to  $x = \sqrt{10} (= 3.162)$ . The point A is given by Nicholson as lying on the curve, and the point B lies on the curve of Schwarzschild.

Trinity College,  
Cambridge.

*The Mode of Propagation of the Sun's Influence in Magnetic Storms.* By the Rev. A. L. Cortie, S.J.

1. In former papers on the relations between magnetic storms and sun-spots (*Monthly Notices R.A.S.*, lxx. pp. 200, 201; lxx. p. 23) it has been pointed out that an area of sun-spot disturbance may be accompanied by several magnetic disturbances as it traverses the sun's disc. Not infrequently such storms occur in batches, sometimes on several succeeding days, the angular separation of the sun-spot and the earth's position in heliographic longitude being at times considerable. The inference from such observations is that the mode of propagation of the influence, which is a condition of magnetic storms, from an area of sun-spot disturbance, is in the form of a set of rays or streamers, and is not restricted to one cathode-like ray discharge. In my last paper (*Monthly Notices R.A.S.*, lxxiii. pp. 431-436), evidence was adduced in three cases, the solar eclipses of 1898, 1905, and 1908, of the existence in the sun's corona of bundles of streamers, diverging apparently from narrow belts of area of sun-spot disturbance, which had been connected with series of magnetic storms during several synodic rotations. In the present paper a similar analysis is made of disturbed areas on the sun's surface, the solar corona, and magnetic storms for the eclipse of 1893 April 16. For the eclipse a very complete set of measures of the positions of the prominences, the coronal arches, and the streamers is contained in Professor Schaeberle's report of the observations of the eclipse made at Mina Bronces, Chile (contributions from the Lick Observatory, No. 4, 1895). For the state of the solar surface the materials used have been the Stonyhurst drawings and the Greenwich Tables, as also a spectroheliographic picture of the prominences and flocculi obtained by Professor Hale on the date of the eclipse at the Kenwood Observatory (*Astronomy and Astrophysics*, 1893, p. 450).

2. The longitude of the central meridian of the sun on the day of the eclipse was  $349^\circ$ , hence the longitude of the E. limb was  $259^\circ$ , and of the W. limb  $79^\circ$ . With regard to the E. limb, although there was no spot actually visible near the limb on the day of the eclipse, there were two spots which were very close:

group 2930 which appeared on April 17, in longitude  $268^{\circ}8$  and latitude  $+24^{\circ}9$ , followed on April 18 by group 2931, longitude  $260^{\circ}2$ , latitude  $+20^{\circ}2$ . These two groups were the return of group 2893, and the former was a regular spot which gradually died out; but the latter, beginning as a small spot, increased its area 58-fold between April 18 and April 25, when it was at a maximum, and developed into a fine disturbance. These two groups were surrounded by extensive and bright faculæ, so that the disturbance included a considerable area just behind the sun's limb on the day of the eclipse. The position angles of the two groups of spots referred to the sun's North Pole were  $65^{\circ}$  and  $70^{\circ}$  respectively. This region is included in that numbered 8 in Professor Schaeberle's catalogue of protuberances, of which he remarks, "a region of numberless protuberances and coronal arches." The region was much disturbed, "the confusion from the numerous interlacings is so great that only the more conspicuous ones can be traced throughout the whole arch." Two of these arches in particular, Nos. 43 and 44, started from a point P.A.  $62^{\circ}$ , very near indeed to one of the spots. There can be no possible doubt as to the close connection between the region of spots and faculæ on the one hand, and that of the prominences and arches on the other. But there do not seem to be any coronal streamers connected with this region of any great extent, or any evidence of rays diverging from the spot-area.

The heliographic latitude of the earth during the passage of the spot region across the sun's disc, between April 18-28, was between  $5^{\circ}$  and  $4^{\circ}$  S. Therefore it would be removed some  $25^{\circ}$  from the sun-spot in heliographic latitude. There was only one magnetic storm, but that a great one, during the passage of the spot across the sun's visible hemisphere. It occurred on April 26, the day after the spot attained its greatest area, and when it was removed  $44^{\circ}$  west of the central meridian and the earth's position.

3. There was a fine set of rays on the E. limb in the solar corona at P.A.  $104^{\circ}$ . In his skeleton diagram of the corona Professor Schaeberle makes these rays converge to an area very near to an area of sun-spots and flocculi figured on Professor Hale's photograph. The sun-spot group in question is 2923, visible from April 12 to April 23, but which was at a considerable distance from the E. limb on the day of the eclipse,  $72^{\circ}$ . However, it is possible that the rays converged to this spot, especially as Professor Schaeberle's diagram was drawn quite independently of any pictures of the sun's surface. There was no magnetic storm connected with this group, nor with its return as group 2964 between May 9 and May 18. The directions of the rays on Professor Schaeberle's diagram are not towards the position of the earth.

With regard then to the E. limb, there does not seem to be much connection between coronal streamers and the associated regions of solar-surface disturbance and magnetic disturbances. However, it is well to bear in mind that a year of maximum solar activity is a year of relatively few magnetic storms, because the

mean latitudes of the spots north and south of the equator are as a rule too high to affect the earth, or, as in the case just discussed, the rays proceeding from the spot-area are at too great an angle to the line joining spot and earth.

4. About  $21^\circ$  from the W. limb, in longitude  $57^\circ 8'$  and high S. latitude  $-28^\circ 6'$ , was group 2919, a growing group of sun-spots which reappeared in the next rotation as group 2956. Its position angle on the date of the eclipse referred to the sun's limb was  $241^\circ$ . This was the mean position of a group of great prominences and associated arches, but there are no streamers in this position on the sun in the corona. The spot-group was surrounded by faculæ. There were no magnetic storms from March 29 to April 26, which covers the period of the transit of this spot-group, from April 8 to April 16.

5. Group 2914 was removed about  $33^\circ$  from the sun's limb, at longitude  $46^\circ 4'$  and latitude  $-8^\circ 4'$ . The group, which was visible from April 5 to April 18, consisted of two round spots at the steady stage of type iv., in extensive and bright faculæ. The following spot of the two had lower latitude than the leader, but gradually rose to the same latitude. Its position angle relatively to the limb of the sun was  $262^\circ$ . During the passage of this spot-group across the sun there were no magnetic storms, but it formed a member of an associated set of solar disturbances from January 16 to May 10, with a revival in activity between September 17 and November 16, which was associated with a series of synodical magnetic disturbances. The spot-groups were as follows:—

| Groups. | 1893.    |          | Longitude.    | Latitude.      |                                     |
|---------|----------|----------|---------------|----------------|-------------------------------------|
| 2819    | Jan. 16  | Jan. 27  | $14^\circ 9'$ | $-13^\circ 1'$ | Recurrent series<br>318, Greenwich. |
| 2847    | Feb. 12  | Feb. 23  | $25^\circ 5'$ | $-10^\circ 4'$ |                                     |
| 2881    | March 10 | March 21 | $42^\circ 0'$ | $-8^\circ 2'$  |                                     |
| 2914    | April 5  | April 18 | $44^\circ 6'$ | $-8^\circ 5'$  |                                     |
| 2959    | May 3    | May 10   | $45^\circ 9'$ | $-7^\circ 5'$  |                                     |
| 3187    | Sept. 17 | Sept. 27 | $49^\circ 3'$ | $-8^\circ 1'$  | Revival.                            |
| 3275    | Nov. 12  | Nov. 16  | $47^\circ 0'$ | $-8^\circ 3'$  |                                     |

The magnetic disturbances, taken from the list in the Stonyhurst yearly report, were as follows:—

| Groups. | 1893.    | Magnetic Disturbances. |          |          |          |          |    |    |          |
|---------|----------|------------------------|----------|----------|----------|----------|----|----|----------|
| 2819    | January  | 18                     | 19       | 20       | 21       | 22       | 23 | 24 | 25       |
|         |          | <i>m</i>               | <i>m</i> |          | <i>m</i> | <i>m</i> |    |    | <i>m</i> |
| 2847    | February | 14                     | 15       | 16       | 17       | 18       | 19 | 20 | 21       |
|         |          |                        | <i>m</i> | <i>m</i> | <i>m</i> | :        |    |    | :        |
| 2881    | March    | 13                     | 14       | 15       | 16       | 17       | 18 | 19 | 20       |
|         |          |                        | <i>m</i> | <i>m</i> | <i>m</i> | :        |    |    | :        |
| 2914    | April    | 9                      | 10       | 11       | 12       | 13       | 14 | 15 | 16       |
|         |          |                        | :        |          | :        | :        |    |    | :        |
| 2959    | May      | 6                      | 7        | 8        | 9        | 10       | 11 | 12 | 13       |
|         |          |                        | <i>m</i> |          | <i>m</i> | :        |    |    | :        |
| Faculæ  | June     | 2                      | 3        | 4        | 5        | 6        | 7  | 8  | 9        |
|         |          |                        |          |          |          | <i>m</i> |    |    | <i>m</i> |

In the eclipse photographs the solar corona at the mean position angle indicated above was marked by a great number of widely diverging rays, indicated by the letters  $\alpha$  at P.A.  $254^\circ$ ;  $\beta$  at  $268^\circ$ , and by  $\pi$  at  $256^\circ$  in Professor Schaeberle's list. This last streamer is the most pronounced of a number of much-inclined and conspicuous streamers which cover about  $20^\circ$  of arc. There is no doubt, therefore, in this case of the connection of sun-spot disturbance, coronal stream rays, and magnetic storm sequences. The objection might, however, be raised that there were no magnetic storms accompanying that particular transit of the spot across the sun's disc which was associated with the rays in the corona, especially too as the earth was placed almost radially in heliographic latitude with the spot. But in Professor Schaeberle's diagram of the coronal streamers it will be seen that their directions made large angles with the line joining spot and earth. Moreover, it is not necessary to suppose that the rays proceeding from a centre of disturbance would be invariable in direction in every rotation of the sun corresponding to successive appearances of the spot. It is much more likely that they would change their directions with every change in the development of the spot-group. Hence magnetic storms might occur in one rotation and not in another, and thus would be explained the intermittence of the magnetic action in an active sun-spot region, in some of its returns, which not unfrequently occurs. Mr. Maunder called special attention to such intermittent action in his lists of magnetic storm sequences connected with spots.

The result of this analysis of the connection of regions of solar disturbance with the coronal stream rays and magnetic storms in the case of the eclipse of 1893 has been on the whole to strengthen the inference drawn from a study of the eclipses of 1898, 1905, 1908, that the mode of propagation from the sun of the influence which conditions magnetic storms is in the form of rays diverging from foci of sun-spot disturbance.

6. It is reasonable to suppose, from the investigations of Dr. Nicholson and others, that such rays are the carriers of negatively charged electrons. As these particles recede into space from the sun they would mutually repel one another, and the result would be the formation of clouds of particles as postulated by Father Sidgreaves in his paper "On the Connection between Solar Spots and Earth-magnetic Storms" (*Memoirs R.A.S.*, liv. pp. 91, 92).

The double curvature in some of the streamers of solar coronæ, giving a trumpet-shaped projection, is evidence of such mutual repulsion. Such clouds would more abound in the sun-spot regions, and might reach large dimensions by the cumulative action of successive accretions. The earth, on account of its low heliographic latitude, would encounter a greater number of these clouds after the maximum of sun-spot activity had passed, when the latitude of sun-spots was declining, which would explain the greater number of magnetic storms recorded at such periods. The entry of the earth into such clouds would precipitate magnetic

storms, not as an actual cause, for, as Professor Schuster has shown, this is impossible, but as an instrumental cause, or necessary condition. Great magnetic storms always begin suddenly, and the violent oscillations of the needles end almost as suddenly, suggesting the passage of the earth through a cloud of electrified particles. The function of these particles might be, as Professor Schuster contends, to make the upper atmosphere a better electrical conductor.

*Stonyhurst College Observatory:*

1913 May 3.

*On the Harmonic Analysis of Sun-spot Relative Numbers.*

By Hisashi Kimura, Assoc. R.A.S. (Plate 20.)

*Introduction.*—Along with the investigation of the motion of the Earth's pole, it became extremely desirable to analyse the relative numbers of Sun-spots, in order to see what relation exists between these two phenomena, in view of the suggestion of Halm.

Till the present time this subject has been attacked by Wolf, Thiele, S. Hirayama, and Schuster. They failed, however, to detect all the periodicities, the combination of which may represent even a general outline of the observed numbers. The main difficulties in the separation of periods, which embarrass every one, are caused by the existence of a great many periods of nearly the same lengths; and by the fact that the total duration of observations is not long enough to contain a whole cycle. Several methods for picking out all periodicities from a given curve have been proposed by Schuster, Kühnen and others, yet they are far from conquering the above difficulties. I decided, therefore, to attack the problem by mere trial and the method of successive approximations.

*Process of Calculation.*—The data employed are wholly limited to the Wolf-Wolfer's yearly numbers given in the *Monthly Review* for April 1902; and for the remaining years, from Part I. of the *Meteorologische Zeitschrift* in every year.

As the base-line, I have taken provisionally 46.1, the total mean value of the annual numbers extending from 1750 to 1911. At first, I have determined the phases and the amplitudes of the 11-year and its half periods, on the supposition that they recur exactly seven times in every 78 years. After subtracting those periods with the thus found amplitudes, phases, and the lengths of 11.13 years and its half, I have tried to find very long periodicities. For doing this, 30-year means were made; and from the result, by inspection, two long periods of about 78 and 52 years were detected, then the three elements for those periods were computed

by the method of least squares. Here it ought to be remarked that the amplitude of every period found from such means of

several years, is reduced by the factor  $\frac{\sin \frac{n\mu}{2}}{n \sin \frac{\mu}{2}}$ , where  $n$  is the

number of years taken to form a mean, and  $\mu$  is the yearly angular variation corresponding to the period. Now, on eliminating these long periods with the corrected amplitudes from the result, after subtraction of the 11-year period, I have remarked the important fact that the residual curve is approximately reversed at two points in the years 1755 and 1875. This shows that all remaining periodicities are sine curves whose zero points lie in these two years; and further, that their commensurability is 240 years. On this assumption I have provisionally formed values for the coming years from 1912 till 1990, viz. for 1912 the value of 1838 but with the opposite sign, and for 1913 that of 1839, and so forth. From such a series of 240 years, we got the following periodicities:—

TABLE I.

| Number of Oscillations<br>during 240 Years. | Period in Years. | Amplitude. |
|---------------------------------------------|------------------|------------|
| 23                                          | 10'44            | 15'0       |
| 24                                          | 10'00            | 14'0       |
| 28                                          | 8'57             | 14'0       |
| 20                                          | 12'00            | 13'6       |
| 29                                          | 8'28             | 10'0       |
| 16                                          | 15'00            | 8'7        |
| 18                                          | 13'33            | 8'7        |
| 12                                          | 20'00            | 6'0        |
| 26                                          | 9'24             | 6'0        |
| 19                                          | 12'63            | 5'4        |
| 32                                          | 7'50             | 5'2        |
| 9                                           | 26'67            | 5'         |
| 27                                          | 8'89             | 5'         |
| 33                                          | 7'27             | 5'         |

Although the resultant of all periods given in the above table agrees fairly with the observed quantities, yet there seems to remain some other considerable ones entirely independent of the above group. Now since the assumed cycle of 240 years is still uncertain, all the elements found above are very rough; yet they have been very useful, so that, without them, I should have failed in accomplishing this work.

On proceeding farther, I employed the method of successive approximation, based only on the material of the 162 years really observed. From the residuals, after subtraction of the 11-year

and the two long periods, means of three 50-year series were made, from which the amplitudes and the phases of periods of 10, 8.33, 16.67, and 12.50 years were determined. Then those four periods were eliminated. Next, from the new residuals, the means of two 60-year series were formed, from which the elements of 20, 30, 12, 8.57, 15 and its half were determined. Again, after having subtracted all the periodicities of sub-multiples of 60 years, the elements 10.5 and 7.0, the residuals were found from 21-year series. In such a way, we picked up all periodicities by degrees, until no other considerable ones could be obtained from the final residuals. Thus the first approximation ended. Here it should be noted that, throughout this work, when the whole number of 162 years was not necessary, the part employed was always taken to end with 1911, so as to avoid the earlier part of comparatively small weight.

The second approximation began with the final residuals of the first approximation, to which the numerical values found in the first approximation for any period were added; then the whole was divided into two (sometimes three) parts, the length of one part being chosen as near as possible an exact multiple of the period in question; from each part the amplitude and phase were calculated by the usual method of least squares, and the correction to the length of period was found from the difference of the phases. If the amplitudes from every part differ too much from each other, it proves that there remain some hidden periods resonating with that period, or that some harmonising false ones were already taken into account. In such cases, careful examination was made, and the discrepancies were removed by addition or subtraction of other periods. We hardly met, however, such a confusion when we were treating periodicities longer than 8 years. Besides, in cases when the corrected length of period differed too much from the original one, although the amplitudes in both parts coincided fairly, a new appropriate length was chosen and the process of calculation was gone through again. And whatever the cause might be, if the new elements of any period differed much from the old ones, the residuals were corrected by the differences of old and new calculations. In such a manner, applying all periodicities successively, according to the order of amplitude, we have obtained more approximate values for the lengths of periods than those first arrived at.

Next, to obtain still closer approximation, a third approximation was carried out just in the same way as in the second, with some alterations in the order of application, for the purpose of eliminating possible systematic errors caused by it.

*Results of Calculation.*—As usual, we represent the relative number of sun-spots by

$$\Sigma A \sin \left( \frac{2\pi}{T} + H \right),$$

where A is amplitude; T, the length of period; H, the phase at the epoch 1835·5. The following table gives the three elements of every period found in the second and third approximations, arranged in the order of the length of period:—

TABLE II.

|    | Third Approximation. |      |       | Second Approximation. |      |       |
|----|----------------------|------|-------|-----------------------|------|-------|
|    | T.                   | A.   | H.    | T.                    | A.   | H.    |
| 1  | 164 <sup>y</sup> *   | 1·2  | 90°0  |                       |      |       |
| 2  | 82·2†                | 18·7 | 358·7 | 80 <sup>y</sup> ·5    | 20·2 | 358°4 |
| 3  | 54·2                 | 10·8 | 79°0  | 53·8                  | 10·3 | 80·3  |
| 4  | 38·91                | 5·3  | 145·2 | 40·                   | 3·6  | 142·4 |
| 5  | 29·95                | 4·7  | 18°0  | 30·                   | 5·8  | 23·7  |
| 6  | 25·63                | 3·7  | 33·7  | 26·95                 | 3·1  | 14·3  |
| 7  | 20·03                | 10·4 | 192·7 | 20·                   | 9·1  | 189·3 |
| 8  | 19·08                | 4·9  | 35°0  | 18·41                 | 4·2  | 48·8  |
| 9  | 16·59                | 3·2  | 50·6  | 17·02                 | 2·6  | 217·2 |
| 10 | 14·97                | 8·0  | 105·1 | 14·99                 | 8·4  | 104·8 |
| 11 | 13·53                | 8·0  | 206°0 | 13·50                 | 6·5  | 209·4 |
| 12 | 12·49                | 3·8  | 233·6 | 12·52                 | 3·7  | 228·4 |
| 13 | 12·05                | 15·1 | 40°1  | 12·04                 | 15·4 | 42·4  |
| 14 | 11·114               | 27·5 | 8·5   | 11·113                | 27°0 | 10°1  |
| 15 | 10·48                | 14·9 | 236·7 | 10·53                 | 12·2 | 244·5 |
| 16 | 9·99                 | 14·3 | 337°0 | 10·03                 | 13·7 | 331·5 |
| 17 | 9·24                 | 7·8  | 47·6  | 9·24                  | 6·6  | 53·6  |
| 18 | 9·02                 | 6·3  | 200°1 | 8·99                  | 5·8  | 163·8 |
| 19 | 8·55                 | 10·6 | 301·5 | 8·55                  | 11·8 | 300·9 |
| 20 | 8·25                 | 9·7  | 33·9  | 8·25                  | 9·8  | 22·4  |
| 21 | 7·53                 | 5·0  | 269·9 | 7·52                  | 5°0  | 261·8 |
| 22 | 7·02                 | 4·9  | 16·5  | { 7·23                | 3°0  | 350°0 |
|    |                      |      |       | { 7·05                | 2·1  | 355·1 |
| 23 | 5·95                 | 5·1  | 70°0  | 5·94                  | 4·8  | 84·2  |
| 24 | 5·74                 | 5·4  | 26·9  | 5·75                  | 4·6  | 27°0  |
| 25 | 5·49                 | 6°0  | 314·3 | { 5·57                | 3·7  | 335·5 |
|    |                      |      |       | { 5·42                | 3·8  | 286·2 |
| 26 | 4·85                 | 4·3  | 11°1  | 4·87                  | 2·5  | 1·9   |
| 27 | 4·35                 | 2·7  | 93·7  | 4·15                  | 2·4  | 3°0   |
| 28 | 4·07                 | 2·8  | 3·1   |                       |      |       |
| 29 | 3·63                 | 3·1  | 244·6 |                       |      |       |

\* Assumed as the double of 82<sup>y</sup>·2.

† This becomes 83<sup>y</sup>·3 if we take the double period out of account, but the existence of the latter is highly probable from several points of view.

A comparison between the observed and the calculated relative numbers of sun-spots is given in Plate 20, diagram I.\*

Diagram II. exhibits the sum of the seven longest periods.

Lastly, I give here a table of periodicities found by several authors; for mine, however, only the corresponding ones are given for brevity.

TABLE IV.

| Author.    | T. N. Thiele.       | R. Wolf.                           | A. Schuster.                                           | S. Hirayama.                                             | H. Kimura.               |
|------------|---------------------|------------------------------------|--------------------------------------------------------|----------------------------------------------------------|--------------------------|
| Reference. | A. N. 1193.         | A. N. 2563.                        | Philos. Trans. of<br>the R.S. of London,<br>vol. ccvi. | Tokyo Sugatu.<br>Buturi Kizi-Gaiyo,<br>vol. lli., No. 9. | Table II.                |
| Origin.    | ..                  | 45°5                               | ..                                                     | 47°8                                                     | 45°1                     |
|            | Period.             | Period. Amp.                       | Period. Ratio<br>of Amp.                               | Period. Amp.                                             | Period. Amp.             |
|            | <sup>y</sup> 11·195 | <sup>y</sup> 11 $\frac{1}{3}$ 21·2 | <sup>y</sup> 11·125 1·00                               | <sup>y</sup> 11·13 43·8                                  | <sup>y</sup> 11·114 27·5 |
|            | ...                 | 81· 17·7                           | ... ..                                                 | ... ..                                                   | 82·2 18·7                |
|            | 9·807               | 10· 17·1                           | ... ..                                                 | ... ..                                                   | 9·99 14·3                |
|            | ...                 | 8 $\frac{1}{3}$ ...                | 8·36 $\frac{1}{3}$                                     | 8·35 8·0                                                 | 8·25 9·7                 |
|            | ...                 | ...                                | 13·5 ...                                               | 13·36 5·9                                                | 13·53 8·0                |
|            | ...                 | ...                                | 5·56 0·24                                              | 5·57 11·8                                                | 5·49 6·0                 |
|            | 5·95                | ... ..                             | ... ..                                                 | ... ..                                                   | 5·95 5·1                 |
|            | ...                 | ...                                | 4·79 $\frac{1}{3}$                                     | ... ..                                                   | 4·85 4·3                 |
|            | 3·76                | ... ..                             | 3·71 0·11                                              | ... ..                                                   | 3·63 3·1                 |
|            | ...                 | ...                                | ... ..                                                 | 16·70 7·1                                                | 16·59 3·2                |
|            | 6·05                | ...                                | ...                                                    | ...                                                      | ...                      |
|            | 5·24                | ...                                | ...                                                    | ...                                                      | ...                      |
|            | 9·74                | ...                                | ...                                                    | ...                                                      | ...                      |
|            | ...                 | ...                                | 2·78 0·09                                              | ...                                                      | ...                      |

## SUMMARY AND CONCLUSIONS.

(1) The 11-year period is not so conspicuous as generally considered, although the most important of all; its length is rather nearer to  $11\frac{1}{6}$  year than  $11\frac{1}{3}$ .

(2) To my surprise, there are a great many periodicities lying between 8 and 12 years, most of them being of considerable relative amplitude.

(3) As to very long periods, besides the Wolf's 81-year, a 54-year period has been detected, and an interesting relation exists between their lengths, namely in the ratio about 3 : 2.

(4) The predicted values for the coming 40 years suggest that, during the first 10 years, the Sun's disc will remain comparatively quiet, and that afterwards spot activity, even at minima, will be greater than the average.

\* [The author's table, showing the contribution of each of the twenty-nine periods, and a comparison of their sum with the observed Sun-spot numbers, is accessible in the Society's Library. Owing to the large number of columns it is not practicable to print it here.—SECS.]

(5) With respect to the degrees of accuracy for the elements of the various periods, I could not give any numerical amounts, but we can, in a greater or lesser degree, judge them by comparing the results in the second and third approximations given in Table II. We do not give much weight to periodicities shorter than 5 years. A critical examination of the final residuals gives clear indications that there yet remain outstanding sensible terms of shorter periods. Of course, the detection of such short periods from the annual means cannot be expected; the monthly means are suitable for the purpose, but they are left for a future occasion.

(6) It cannot be considered that all the periods in Table II. are entirely independent of each other. They may be grouped according to their mutual relationships as follows:—

| 162 years.          | 78 years.          | 60 years.         | 50 years.         |
|---------------------|--------------------|-------------------|-------------------|
| 164' × 1 = 164'     | 38'91 × 2 = 77'82  | 29'95 × 2 = 59'90 | 16'59 × 3 = 49'77 |
| 82'2 × 2 = 164'4    | 25'63 × 3 = 76'89  | 20'03 × 3 = 60'09 | 12'49 × 4 = 49'96 |
| 54'2 × 3 = 162'6    | 19'08 × 4 = 76'32  | 14'97 × 4 = 59'88 | 9'99 × 5 = 49'95  |
| 13'53 × 12 = 162'36 | 11'114 × 7 = 77'80 | 12'05 × 5 = 60'25 | 8'25 × 6 = 49'50  |
| 9'02 × 18 = 162'36  | 4'85 × 16 = 77'60  | 9'99 × 6 = 59'94  | 7'02 × 7 = 49'14  |
|                     | 4'07 × 19 = 77'33  | 8'55 × 7 = 59'85  | 5'49 × 9 = 49'41  |
|                     |                    | 7'53 × 8 = 60'24  |                   |
|                     |                    | 5'95 × 10 = 59'50 |                   |
|                     |                    | 5'49 × 11 = 60'39 |                   |

(7) The important question respecting the possibility of the direct influence of the solar energy upon terrestrial phenomena, such as latitude variation and the changes of the meteorological factors, etc., cannot be solved till similar analyses of the latter have been made. It is, however, already known to me that latitude variation does contain some of the same long-periods as the Sun-spots. As is also Schuster's opinion, I have believed for a long time that such common periodicities among the Sun-spots and the latitude variation do not come from the direct influence of the former on the latter, but from some common perturbing causes, such as cosmic matter, which may affect all bodies of our system. Indeed, the present paper may perhaps serve as a clue in the future investigation of the above problem.

My sincere acknowledgments are due to Professor Shin Hirayama, who has informed me kindly and liberally about all reference papers, which he had searched during his investigation respecting this subject; and I have also to thank Mr. T. Mori for aid in the very laborious calculations.

*Mizusawa International Latitude Observatory:*  
1913 March 12.

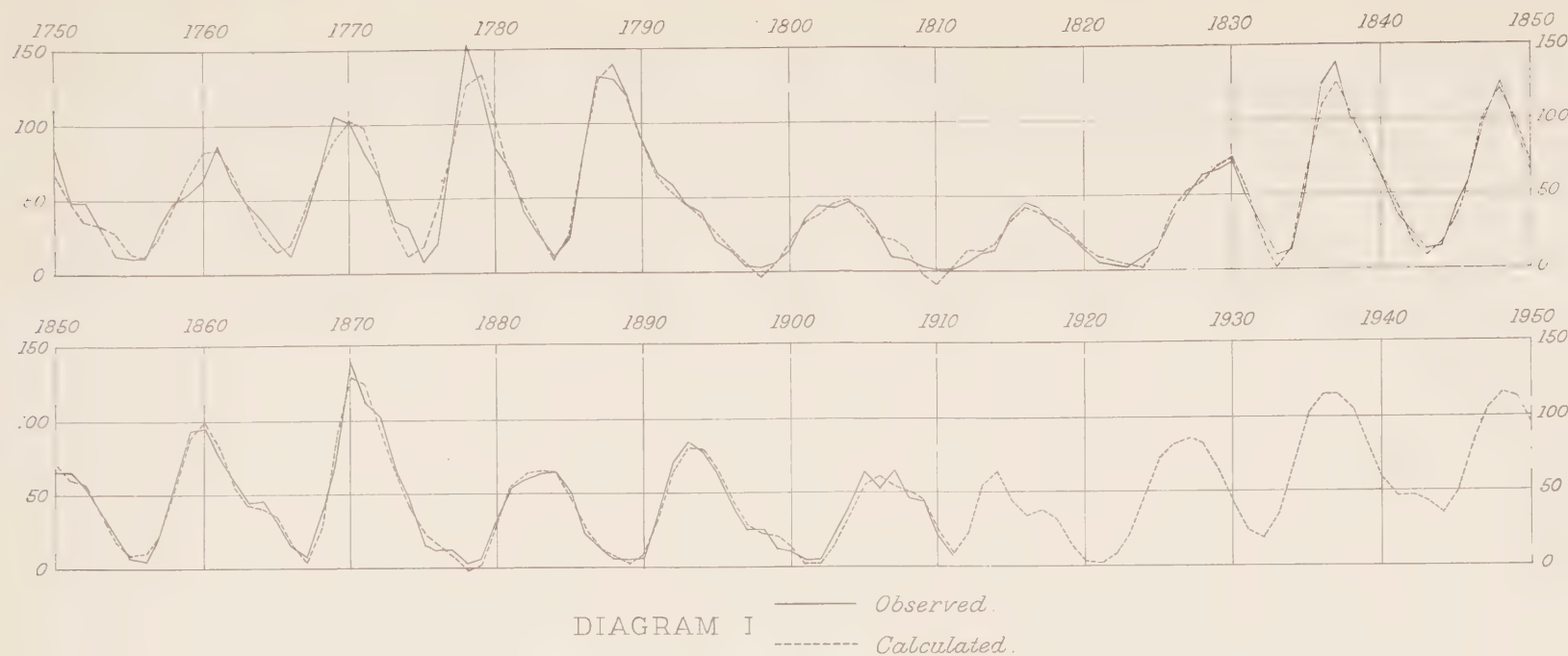


DIAGRAM II Resultant of seven long periods, 164, 82.2, 54.2, 38.91, 29.95, 25.63, 20.03, years.

H. KIMURA. HARMONIC ANALYSIS OF SUN-SPOT RELATIVE NUMBERS.

West, Newman lith.



*On the Harmonic Analysis of Wolf's Sun-spot Numbers, with Special Reference to Mr. Kimura's Paper. (Papers of the I.U.S.R. Computing Bureau, No. IX.)* By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Mr. Kimura's paper was of special interest to me because I had been working at the same problem as a sequel to the comparison of the Sun with S Persei (*Monthly Notices*, lxxiii. p. 116). In the paper cited, I used the periodogram analysis of the Sunspot numbers tabulated by Professor Schuster (*Phil. Trans.*, vol. 206, A, p. 69), but it seemed possible that the various contributing periodicities might be more completely disentangled than appears in that paper, and I had made some progress in that direction when Mr. Kimura's paper was received, containing (as printed above) what purports to be a complete identification of a number of periodicities, and also (what is not printed) a comprehensive table showing the application of these periodicities, with the calculated coefficients to satisfy the observations within narrow limits. This table is naturally a long one: it would occupy more than a dozen pages of the *Monthly Notices*, and in such cases it is usual for some member of Council to examine and report upon the desirability of undertaking the printing. At my own request, this examination was entrusted to me, as one working in the same field. I found the paper of engrossing interest, and have spent a large part of the last month in work arising out of it. This work has led me to the conclusion that for the present it is advisable not to print the long table, for reasons given below. But at the same time I hope to show that points of great importance are raised by Mr. Kimura's investigation, on which more light is desirable: and as my conclusions may be wrong, I desire to take full responsibility for them. Hence I venture to preface these few words explaining the circumstances.

2. By a series of steps, which need scarcely now be retraced in detail, I ultimately arrived at the conclusion that Mr. Kimura's analysis was an approximate equivalent for one which can be stated in simpler terms. He has shown how his periods can be grouped in periods of 162, 78, 60, and 50 years. Study of his figures led me, firstly, to a control period of 300 years (of which 60 and 50 are exact, and 162 and 78 approximate sub-multiples), and later to a control period of about half this length, ultimately adopted as 156 years, which has some convenient factors ( $3 \times 4 \times 13$ ). If now we form the successive sub-multiples of 156 years by the series 1, 2, 3, 4, etc., it will be seen from the comparison given in Table I. that Mr. Kimura's periods are approximately reproduced. Further, when the coefficients for the terms of this regular Fourier sequence are calculated, they resemble those obtained by Mr. Kimura.

3. It seems to me significant that Mr. Kimura, starting in such a very different manner, as his paper explains, should have been led so close to the Fourier sequence: the suggestion is that the

Fourier sequence is the proper expression for the results, and that though others may be found, they will tend to approach the Fourier sequence. Before comparing the two methods of representation, let us first consider some characteristics of the Fourier sequence. At first sight there is something arbitrary about the period of 156 years, which militates against the idea of a unique representation. But the choice is really not arbitrary. A Fourier sequence only applies to a series which will recur, so that the beginning and end must fit, not only in the actual Wolf's number, but in the slope and curvature, etc. It is useless to find Fourier's sequence unless we take a bit of the curve near 1750 and match it as well as possible with the piece near 1906. This was done, and the period 156 years was closely indicated. Being otherwise convenient (owing to the factors  $12 \times 13$ ) it was adopted for use.

TABLE I.

| Divisor<br>of 156. | Period<br>in Years. | Kimura<br>Period. | Differ-<br>ence.   | Coefficient.<br>T. Kim. |      | Divisor<br>of 156. | Period<br>in Years. | Kimura<br>Period. | Coefficient.<br>T. Kim. |     |
|--------------------|---------------------|-------------------|--------------------|-------------------------|------|--------------------|---------------------|-------------------|-------------------------|-----|
| 1                  | 156 <sup>y</sup> ·0 | 164 <sup>y</sup>  | +8 <sup>y</sup> ·0 | 2·5                     | 1·2  | 28                 | 5·58                | 5·49              | 4·6                     | 6·0 |
| 2                  | 78·0                | 82·2              | +4·2               | 17·9                    | 18·7 | 29                 | 5·38                |                   |                         |     |
| 3                  | 52·0                | 54·2              | +2·2               | 10·6                    | 10·8 | 30                 | 5·20                | ...               | 1·1                     |     |
| 4                  | 39·0                | 38·91             | -0·1               | 4·2                     | 5·3  | 31                 | 5·03                |                   |                         |     |
| 5                  | 31·2                | 29·95             | -1·2               | 3·1                     | 4·7  | 32                 | 4·88                | 4·85              | 3·8                     | 4·3 |
| 6                  | 26·02               | 5·63              | -0·4               | 6·7                     | 3·7  | 33                 | 4·73                | ...               | 2·1                     |     |
| 7                  | 22·3                | 20·03             | -2·3               | 2·9                     | 10·4 | 34                 | 4·59                |                   |                         |     |
| 8                  | 19·5                | 19·08             | -0·42              | 5·8                     | 4·9  | 35                 | 4·46                |                   |                         |     |
| 9                  | 17·3                | 16·59             | -0·71              | 3·9                     | 3·2  | 36                 | 4·33                | 4·35              | 2·1                     | 2·7 |
| 10                 | 15·6                | 14·97             | -0·63              | 6·2                     | 8·0  | 37                 | 4·22                |                   |                         |     |
| 11                 | 14·2                | 13·53             | -0·67              | 6·4                     | 8·0  | 38                 | 4·10                |                   |                         |     |
| 12                 | 13·0                | 12·49             | -0·51              | 7·6                     | 3·8  | 39                 | 4·00                | 4·07              | 1·7                     | 2·8 |
| 13                 | 12·0                | 12·05             | +0·05              | 15·0                    | 15·1 | 40                 | 3·90                | ...               | 0·8                     |     |
| 14                 | 11·14               | 11·11             | -0·03              | 25·8                    | 27·5 | 41                 | 3·81                |                   |                         |     |
| 15                 | 10·40               | 10·48             | +0·08              | 14·0                    | 14·9 | 42                 | 3·72                | ...               | 1·0                     |     |
| 16                 | 9·75                | 9·99              | +0·24              | 13·6                    | 14·3 | 43                 | 3·63                | 3·63              | ...                     | 2·1 |
| 17                 | 9·20                | 9·24              | +0·04              | 7·5                     | 7·8  | 44                 | 3·55                | ...               | 1·5                     |     |
| 18                 | 8·67                | 9·02              | +0·35              | 10·4                    | 6·3  | 45                 | 3·47                | ...               | 1·4                     |     |
| 19                 | 8·21                | 8·55              | +0·34              | 10·6                    | 10·6 | 46                 | 3·39                |                   |                         |     |
| 20                 | 7·80                | 8·25              | +0·45              | 2·2                     | 9·7  | 47                 | 3·32                |                   |                         |     |
| 21                 | 7·43                | 7·53              | +0·10              | 5·3                     | 5·0  | 48                 | 3·25                | ...               | 1·3                     |     |
| 22                 | 7·09                | 7·02              | -0·07              | 7·2                     | 4·9  | 49                 | 3·18                |                   |                         |     |
| 23                 | 6·78                | ...               | ...                | 2·9                     | ...  | 50                 | 3·12                |                   |                         |     |
| 24                 | 6·50                | ...               | ...                | 0·4                     | ...  | 51                 | 3·06                | ...               | 3·9                     |     |
| 25                 | 6·24                | ...               | ...                | 1·0                     | ...  | 52                 | 3·00                | ...               | 0·3                     |     |
| 26                 | 6·00                | 5·95              | -0·05              | 3·7                     | 5·1  | 53                 | 2·94                |                   |                         |     |
| 27                 | 5·78                | 5·74              | -0·04              | 5·1                     | 5·4  | 54                 | 2·89                | ...               | 1·6                     |     |

4. But 156 years is also approximately the whole length of the series of observations. Now, it is well known that *any* series of numbers can be represented by a Fourier series of this kind. Having determined the coefficient of each term and its phase, we can add together the items to reproduce the original series of observations; and the fact that the original series is so reproduced is, taken alone, a proof of nothing more than the correctness of the arithmetic. The total number of terms being theoretically infinite, we have theoretically to calculate an infinite number of terms: but those of short period can only have sensible coefficients if the series is very irregular, *i.e.* if consecutive terms are liable to differ largely. With a fairly regular series, such as Wolf's Sun-spot numbers, the coefficients of the small terms will be small; and we may stop at any convenient point with the confident expectation that the residuals, due to neglect of smaller periods, will be small.

5. But a further important characteristic of the simple Fourier series is that each term is completely independent of every other, since even two adjacent terms run through every phase relatively to one another. Each term gains or loses one complete period on those next to it, and an integral number of complete periods on other terms. If, however, we select a period intermediate between any of those specified in the second column, we shall get contributions not only from the terms on either side of it, but from all the other terms. Thus, if we take a period of 10.71 years (which happened to be tried in the course of the work), we shall get a mixture not only of the 10.40 and 11.14 terms, but of all other terms as well, since no single one of them runs through an exact number of periods relatively to 10.71 years. But, of course, the contributions of distant terms are small; it is only neighbouring terms which make sensible contributions. Nevertheless, the total result may not be easy to anticipate. Take, for instance, the case already mentioned of 10.71 years. The coefficients of the terms next it are 25.8 and 14.0: and we might expect something intermediate for the intermediate period. But the coefficient comes out 8.2, less than either. On the other hand, if we take a period of 11.54 years, on the other side of the 11.1-year period, and about half way to the next greater, we get a coefficient 23.3 nearly equal to the 11.1 period itself. This would suggest that the maximum coefficient will be found at some period intermediate between 11.54 and 11.14 years: in spite of the fact that Mr. Kimura obtains a coefficient greater than any of these for a period slightly *less* than 11.14 years.

6. These two instances will suffice to show that it is not easy to predict from general considerations what will be the effect of any departure from the strict Fourier sequence. It is therefore with some diffidence that I put forward the suggestion that Mr. Kimura's terms are the approximate equivalent of the Fourier sequence: but the equivalence is probably close enough to put the two series under the same disability, *viz.* the fact that they satisfy the observations in itself proves nothing more than that the

arithmetic has been done correctly. It seems, moreover, probable that any number of such equivalents could be constructed, all departing more or less from the strict Fourier sequence, and all satisfying the observations equally well: from this fact alone they would have equal claims on our attention.

7. We now have to inquire, however, how far the coefficients themselves furnish any evidence of physical realities. It will be seen from the table (perhaps more readily from a diagram) that there is a methodical run about the coefficients which at first encouraged me to believe that something real was indicated. But further consideration led to the conclusion that this methodical run proved nothing more than that the series of Wolf's numbers is fairly regular, and has no discontinuities or abruptnesses. The study of the Fourier sequence, however, has raised some questions of general interest, such as:

- (a) Whether in constructing a periodogram we ought not always to begin with a Fourier sequence.
- (b) Whether, having done this, we cannot practically infer the whole periodogram.

Incidentally also Mr. Hirayama's records of Sun-spots in Chinese Annals (*Observatory*, vol. xii.) were studied, and were fitted with some exactness to modern results. But the examination of these points is not complete, and must be reserved for a future paper.